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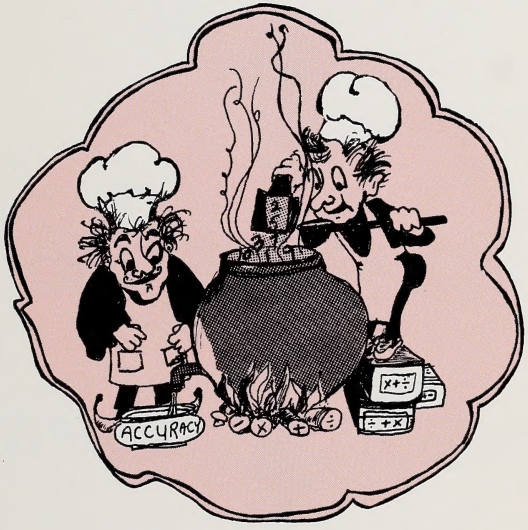
PRACTICAL MATHEMATICS



WHOLE NUMBERS

module 1

ALBERTA CORRESPONDENCE SCHOOL
ALBERTA EDUCATION
BARRHEAD, ALBERTA

Practical Mathematics

Module 1

WHOLE NUMBERS

Practical Mathematics
Student Module
Module 1
Whole Numbers
Alberta Correspondence School
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IMPORTANT!

PLEASE READ THIS INTRODUCTION BEFORE BEGINNING YOUR FIRST LESSON.

Content and Purpose of Course

In its entirety, this Practical Mathematics Series consists of eight modules, each containing from four to seven lessons. The titles of these modules are as follows:

Module 1: Whole Numbers
Module 2: Fractional Numbers
Module 3: Decimal Numbers
Module 4: Ratio and Proportion
Module 5: Measurement
Module 6: Introductory Algebra
Module 7: Intermediate Algebra
Module 8: Trigonometry

Taken individually, these modules should provide a person with instruction and practice in specific areas in which he finds that his mathematical background is weak.

The first five modules, taken as a unit, should equip a person to handle most practical mathematical problems encountered in day-to-day living. The next two modules cover the basics of algebra required for subsequent vocational mathematics courses. The last module covers the basics of trigonometry.

The series, Practical Mathematics, was also developed with the needs of prospective tradesmen in mind. Modules 1-5 inclusive were designed to prepare students mathematically for certain apprenticeship programs. In other words, if a person completes the first five modules of this series, he should be prepared to write the mathematics section of the examination set by the Apprenticeship Board for entrance into the following apprenticeship programs:

Agricultural Mechanic
Appliance Serviceman
Auto Body Mechanic
Baker
Bricklayer
Carpenter
Cook
Floorcovering Mechanic
Gasfitter
Glassworker
Heavy Duty Mechanic
Insulator
Lather
Machinist
Millwright
Motor Mechanic
Painter and Decorator
Partsman
Plasterer
Plumber
Refrigerator Mechanic
Roofer
Sheet Metal Mechanic
Steamfitter
Steel Fabricator
Tile Setter
Welder

If a person completes Modules 1 to 7 inclusive, he should be prepared to write the Apprenticeship Board's mathematics examination for entrance into the apprenticeship programs listed below:

Communication Electrician	Instrument Mechanic
Electrician	Power Electrician
Electrical Mechanic	Radio Technician

There is no prescribed textbook for this course. All the explanatory material and exercises are provided in the lessons.

Self-Correcting Exercises

A number of self-correcting exercises appear in the lessons in this course. You should work through each of these exercises carefully and when you have completed the **entire** exercise, check your answers with those provided at the end of the lesson. Correct any errors that you have made in these exercises. If you do not understand some of the answers, ask your correspondence teacher about them. It is important that you handle these self-correcting exercises properly as they help prepare you for the assignments that follow.

Inventory Tests

Inventory tests appear at the beginning of Modules 2-5 inclusive. These lessons are designed to test the student's grasp of basic skills which should be mastered before additional lessons are attempted. Complete each inventory test and send it in for correction before beginning any other lessons in that module.

Optional Sections

Optional sections occur in some lessons in this course. Students not progressing beyond Module 5 are not required to complete these sections, but may do so if they wish. Students who plan on taking Modules 6, 7 and 8 **must** complete these optional sections.

Dear Student:

As of June 1, 1989 you will not be required to send in any lessons in Practical Mathematics (Modules 1 - 5). At the back of the module you will find a booklet of answers and solutions.

Study the lesson notes very carefully, work on the exercises and then check your work by using this booklet of answers and solutions. If, however, you experience difficulty in doing some questions you may telephone the Alberta Correspondence School at 674-5333. Students who reside in Alberta may call the school free of charge through the local R.I.T.E. number in your area or the Zenith number 22333 (if there is no R.I.T.E. centre).

When you have completed your home study and feel confident that you know the material then you may proceed onto Mathematics 14 (if you require credits) or into Practical Mathematics (Modules 6, 7 and 8) or Basic Algebra and Geometry (if you require an academic mathematics course). However, if you are undecided which route to take then write us a letter or telephone us by using the above number.

We wish you success in this course.

Ronald Billey

Ron Billey
Vice-Principal
Mathematics Science Department

MODULE 1: WHOLE NUMBERS

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Practical Mathematics
Module 1
Lesson 1

READING AND WRITING WHOLE NUMBERS

Numbers can be very useful — we use them daily in our routine activities. Have you ever had a day where you got out of bed 37 minutes late and only had 23 minutes to get ready for work? So in your rush to save time you only cooked your 7-minute egg for 3 minutes and forgot to put out 4 milk bottles for the milkman. In your last desperate attempt to arrive at work on time you drove 65 km/h (kilometres per hour) in a 50 km/h zone and picked up a 30 dollar speeding ticket. And as if your morning wasn't already ruined you arrived at work 13 minutes late and received a 10 minute lecture from your boss on being punctual for work in the mornings.

Name 2 incidents in your routine activities today in which you had to use numbers.

_____

_____

_____

Everyone uses numbers. It is for this reason that we must gain a solid understanding of numbers and how to use them in counting, comparing, and computing.

Topic One: Naming Numbers

In this lesson we will begin to take a closer look at the whole numbers that you are already familiar with in counting.

If you were asked to start at 1 and begin counting, what is the largest number that you could name? For any number that you name, a greater number can always be obtained by adding 1 to your number. Therefore, we can never really name the largest possible number. The counting numbers go on and on without end; we say that they are INFINITE.

We can express a certain number idea by a variety of symbols. For example, the following symbols all represent the number idea "three".

EXAMPLE: 3 2+1 7-4 three

3×1 $12 \div 4$ $\frac{18}{6}$

We are fortunate today that our counting is based on the Hindu-Arabic system of numeration. (A numeration system is simply a method of representing numbers.) This system has several distinct advantages not found in other numeration systems.

The Hindu-Arabic system makes use of ten basic symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. These ten DIGITS can be used to write any number, no matter how large an idea it represents.

EXERCISE - NAMING NUMBERS

1. Give four other symbols that could be used to represent each number idea below.

a. 12	b. 9	c. 5×9
$10+2$		

2. a. What two numbers can you name by using only the digits 5 and 7?

   b. How many 3 - digit numbers can be named by using only the digits 4, 7 and 9? Write them.

   c. What is the <u>smallest</u> 3-digit number than can be named if one of the digits is an 8 and each digit can only be used once?

   d. If all digits are the same, what is the <u>largest</u> 3-digit number that can be named?

   e. What does it mean to say that the counting numbers are infinite?

Topic Two: Place Value

You have used the basic ten digits to write numbers greater than 9. When writing numbers in the Hindu-Arabic system, we arrange the digits so that they have PLACE VALUE. The value represented by each digit in a number is determined by the place that digit occupies in the numeral.

The digit 0 was invented by the Hindus and is used as a PLACEHOLDER in any column not having a numerical value. Using 0 as a placeholder is especially important when reading and writing numbers.

A. Three-Digit Numerals

Our numeration system is also known as the DECIMAL SYSTEM, because we group by tens. (The prefix "deci-" comes from the Latin language and means "tens".)

EXAMPLE: To illustrate place-value let's look at the number 872 and name the place value held by each digit. Always start at the right-hand side of the number.

HUNDREDS	TENS	ONES
8	7	2

Thus the digits in the number 872 have the values:

2 ones
7 tens
8 hundreds

Self-Correcting Exercise #1

Answers may be found on page 17 of this lesson. Once you have done the exercise check your own work. Correct all errors that you make.

1. Read the numeral. Then tell how many hundreds, tens, and ones.

   a. 427 _____ hundreds _____ tens _____ ones

   b. 840 _____ hundreds _____ tens _____ ones

   c. 709 _____ hundreds _____ tens _____ ones

   d. 87 _____ hundreds _____ tens _____ ones

   e. 555 _____ hundreds _____ tens _____ ones

2. Name the place value held by each digit described below.

   a. In the number 793, the digit 9 stands for 9 *tens*.

   b. In the number 238 the digit 2 stands for 2 _____.

   c. In the number 515 the digit 5 on the left stands for 5 _____.

   d. In the number 702 the digit 0 stands for 0 _____.

3. a. Which number given at the right uses the digit 0 as a placeholder in the tens column? _____

   b. Which number given at the right uses the digit 0 as a placeholder in the ones column? _____

   940

   24

   702

   8

   317

B. Thousands

Just as a growing family needs a larger house, increasing the size of numbers means we must make room for more digits.

EXAMPLE: Let's look at the number 315,702 and name the place value held by each digit.

HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES
3	1	5	7	0	2

← Learn the names of these places.

Thus the digits in the number 315,702 have the values:

2 ones
0 tens
7 hundreds
5 thousands
1 ten-thousand
3 hundred-thousands

Notice how the digits in base-ten numerals are grouped in threes, going from the right to left. Commas are used to separate the groups. Each of the groups is called a PERIOD and each period has a special name. Only the period farthest to the left in a numeral can contain less than three digits.

In the numeral 315,702 the digits 315 belong to the THOUSANDS PERIOD. The digits 702 belong to the UNITS PERIOD.

THOUSANDS			UNITS		
HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES
3	1	5	7	0	2

EXERCISE - THOUSANDS

1. Use commas to separate the following numerals into periods. Be sure to work from the right to left. (The first one has been done for you.)

93,768	773001
422930	7899
209530	66002

2. Fill in the blanks.

   a. Consider the number 903,672

      (i) The digits 903 are in the _____ period.

      (ii) The digit _____ is in the hundreds place.

      (iii) The digit _____ is used as a _____ in the ten-thousands place. (Refer back to page 3.)

   b. Consider the number 279,052

      (i) To divide the number into periods we have placed a _____ between the digits _____ and _____.

      (ii) The digits _____ are in the units period.

      (iii) The digits 79 are in the last two places of the _____ period.

C. Millions

Of course, thousands aren't the largest numbers that we use — we could never become millionaires if we only worked with thousands! You will remember that the basic ten digits can be combined to form any number, no matter how large.

To the left of the thousands period we find the MILLIONS period. Study the examples given below. Notice the names of the individual place-values within each period.

Memorize the names of these places. →

	MILLIONS			THOUSANDS			UNITS		
	HUNDRED MILLIONS (100,000,000)	TEN MILLIONS (10,000,000)	MILLIONS (1,000,000)	HUNDRED THOUSANDS (100,000)	TEN THOUSANDS (10,000)	THOUSANDS (1,000)	HUNDREDS (100)	TENS (10)	ONES (1)
1.	2	7	6	3	8	0	4	5	9
2.			4	6	0	0	1	5	7

Special values are assigned to each of the individual places in a numeral. Study the place values given in the previous chart. Notice that each place has a value ten times as great as the next place on the right. The value of any particular digit in a numeral can be found by multiplying the digit by the value of the place it holds. The sum of the values of the particular digits is what the numeral represents.

1. The first number in the chart is 276,380,459.

   (i) The digit 2 is in the hundred-millions place. It represents the number (2 × 100,000,000) or 200,000,000.

   (ii) The digit 6 is in the _____ place. It represents the number (6 × 1,000,000) or _____.

   (iii) The digit 8 is in the _____ place. It represents the number (_____ × 10,000) or _____.

2. The second number in the chart on page 7 is 4,600,157.

(i) The digit _____ is in the millions place. It represents the number (_____ × 1,000,000) or _____.

(ii) The digits _____ are in the thousands period. (Read this one carefully! THINK!)

(iii) The digit _____ is in the thousands place. It represents the number (_____ × 1,000) or 0.

(iv) The digit 6 is in the _____ place. It represents the number (_____ × _____) or _____.

(v) Find the sum of the values of the individual digits by completing the chart below.

Digit	Value
4	4,000,000
6	_____
0	0
0	_____
1	100
5	_____
7	_____
	_____ ← sum

Is this sum the same as the given number? _____

D. Numbers Larger Than Millions

Although we don't often use numbers larger than millions, it is interesting to note the names of the periods to the left of the millions period. You should be aware of the billions and trillions period. It is not important to remember the names of periods larger than a trillion but don't forget that they do exist.

If we divide a number into periods starting from the right and moving to the left, the names of the periods are: units, thousands, millions, billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions and decillions.

EXERCISE - PLACE VALUE

1. Matching. Think about the place value of digits. In the blanks before each description, write the letter of a numeral from the list on the right that fits the description. Any numeral may be used more than once and some may not be used at all. The first one is done for you.

(i) __c__ 2 is in the tens place

(ii) _____ 9 is in the millions place

(iii) _____ 0 is used as a placeholder in the ones place

(iv) _____ there are 4 hundred-millions

(v) _____ there are 7 ten-thousands

(vi) _____ there is only 1 million

(vii) _____ 2 is in the thousands place

(viii) _____ there are 6 hundreds

(ix) _____ 6 is in the thousands place

(x) _____ there are no hundreds

a. 45,994,575

b. 97,972,266

c. 105,710,620

d. 129,285,046

e. 431,669,275

f. 654,697,361

g. 988,606,705

2. Give the value of each position described below. The first one is done for you. (The chart on page 7 may help you.)

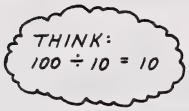
a. The position immediately to the left of the hundreds place. thousands OR 1,000

b. The position immediately to the right of the hundreds place. _____

c. The first position on the extreme right of any number. _____

d. The position 2 places to the left of the thousands place. _____

3. Think carefully before you answer.



a. How many tens make a hundred? 10

b. How many tens make a thousand? _____

c. How many hundreds make a thousand? _____

d. How many thousands make a million? _____

e. How many hundreds make ten-thousand? _____

Topic Three: Reading and Writing Numerals

In this lesson you have studied the use of commas to separate a numeral into periods. When separating a numeral into periods, remember to always work from right to left. On page 8 you were given the names assigned to the different periods. Know these names up to the trillions period.

When we read a base-ten numeral, we give the digits in the period farthest to the left and then give the name of the period. We do the same for the other periods until we reach the period farthest to the right. No name is attached to this period.

	Billions	Millions	Thousands	Units
1				865
2			725	915
3		883	426	912
4	584	903	527	661

1. The first numeral in the chart above is 865. If there is only one group of three digits, no special name is required. The name of this period is units. The number could be read "eight hundred sixty-five" units or just "eight hundred sixty-five".

2. The second numeral in the chart above is 725, 915.

What digits are in the thousands period? _____.

In the units period? _____.

This number is read, "725 thousand, 915 " or " seven hundred twenty-five thousand, nine hundred fifteen".

3. The third numeral in the chart on page 10 is 883,426,912.

What digits are in the billions period? _____

In the units period? _____

In the millions period? _____

In the thousands period? _____

This number is read: "883 _____, 426 _____, 912."

or "Eight hundred eighty-three_____, four hundred

twenty-six _____, nine hundred twelve."

4. The fourth numeral in the chart is _____. What digits are in the billions period? _____. In the thousands period? _____. In the units period? _____. In the millions period? _____. This number is read:

"584 _____, 903 _____, 527 _____, 661."

Remember, when we are writing a numeral in words a hyphen is used with compound numbers from twenty-one to ninety-nine.

Self-Correcting Exercise #2

Answers may be found on page 17 of this lesson. Be sure to spell each word correctly, and remember the use of hyphens in compound words.

1. Write each of the following numerals, dividing each into periods by using commas. Then show how each is read. The first one is done for you.

a. 8234175436 8,234,175,436

8 billion, 234 million, 175 thousand, 436.

Eight billion, two hundred thirty-four million, one hundred seventy-five thousand, four hundred thirty-six.

b. 6560867581 _____

c. 6594126820 _____

d. 431669275 _____

e. 97972266 _____

Sometimes you must translate a number idea from words into digits. Use the following procedure:

1. Pick out the names of the various periods and note the digits that must appear in these periods.

2. Use a zero if no other digit is given to hold a particular place in a number.

3. Use a comma after the digits in each period.

EXAMPLE: Use digits to write the number "forty-six billion, three hundred fifty-eight million, twenty-seven thousand, four."

Solution

Pick out the various periods and note the digits that must appear in these periods. Rewrite the number as 46 billion, 358 million, 27 thousand, 4

The digits 46 must appear in the billions period.

46,■■■,■■■,■■■

The digits 358 must appear in the millions period.

46, 358,■■■,■■■

The digits 27 must appear in the last two places of the thousands period. A zero must be used to hold the place of the hundred thousands.

46, 358, 027,■■■

The digit 4 must appear in the last place of the units period. Zeros must be used to hold the place of the hundreds and the tens.

46, 358, 027, 004

Therefore, the numeral is 46,358,027,004.

Self-Correcting Exercise #3

Answers may be found on page 18 of this lesson.

1. Fill in the blanks.

(a) Consider the number "seventy-four million, ninety-one."

(i) The digits 74 are in the _____ period.

(ii) The digits 91 are in the last two places of the _____ period.

(iii) The number written in digits is _____. Zeros have been used as placeholders in the entire _____ period and in the _____ place.

(b) Consider the number "six billion, seventy-six million, four thousand, three hundred nineteen".

(i) The digit _____ is in the billions period.

(ii) The digits 76 are in the last two places of the _____ period.

(iii) The digit _____ is in the _____ place of the thousands period.

(iv) The digits _____ are in the units period.

(v) The number written in digits is _____. Zeros have been used as placeholders in the _____ place, the _____ place and the _____ place.

2. Use digits to write each of the following numbers.

(a) Fifty-three million, ninety-two thousand, fourteen.

_____

(b) Seventy billion, four million, nine hundred thousand, twelve.

_____

(c) Four million, two thousand.

_____

(d) Fifteen million, forty thousand, six hundred.

_____

(e) Three hundred million, nine hundred six.

_____

EXERCISE - READING AND WRITING NUMERALS

1. Write in words the name of each of the numbers. Be sure to spell correctly, and check the use of hyphens and commas. The first one is done for you.

   a. 76,871,462

   seventy-six million, eight hundred seventy-one thousand, four hundred sixty-two

   b. 2,073

   c. 296,672,000,021

   d. 177,272,609,117,200

   e. 76,998,203

   f. 422,993,003,277

2. Use digits to write each of the numbers.

   a. ninety-two million, seven hundred thirty-nine thousand, six hundred eleven. 92,739,611

   b. thirty billion, four hundred twenty million, two hundred thousand, one. _____

   c. seventy-eight million, ten. _____

   d. eight hundred two million, five hundred forty-two thousand. _____

   e. sixty million, four hundred twelve thousand, forty. _____

   f. two hundred billion, two hundred million, two thousand, twenty. _____

3. Look at the digits that are underlined in each numeral. Tell the values that they represent. The first one is done for you.

   a. 739,<u>563</u>,827 five hundred sixty-three thousand

   b. <u>245</u>,735,902 _____

   c. <u>7</u>,354,326 _____

   d. 98,413,<u>604</u> _____

   e. <u>88</u>,000,000 _____

   f. 421,673,<u>0</u>00 _____

   g. 99,4<u>0</u>6,356 _____

4. Think carefully before you answer. Write the number that is:

   a. One hundred more that 3,792,847. THINK: 3,792,847 + 100 = 3,792,947 3,792,947

   b. Ten thousand more than 3,792,847. _____

   c. Seven hundred thousand more than 3,792,847. _____

   d. Three million less than 3,792,847. _____

   e. Ten less than two thousand. _____

   f. One thousand more than 989. _____

Key to Self-Correcting Exercises in Lesson 1

Exercise #1, page 4

1. a.

427	4	hundreds	2	tens	7	ones
840	8	hundreds	4	tens	0	ones
709	7	hundreds	0	tens	9	ones
87	0	hundreds	8	tens	7	ones
555	5	hundreds	5	tens	5	ones

2. a. tens
   b. hundreds
   c. hundreds
   d. tens

3. a. 702
   b. 940

Exercise #2, page 11

1. a. 8,234,175,436
   8 billion, 234 million, 175 thousand, 436
   Eight billion, two hundred thirty-four million, one hundred seventy-five thousand, four hundred thirty-six.

   b. 6,560,867,581
   6 billion, 560 million, 867 thousand, 581
   Six billion, five hundred sixty million, eight hundred sixty-seven thousand, five hundred eighty-one.

   c. 6,594,126,820
   6 billion, 594 million, 126 thousand, 820
   Six billion, five hundred ninety-four million, one hundred twenty-six thousand, eight hundred twenty.

   d. 431,669,275
   431 million, 669 thousand, 275
   Four hundred thirty-one million, six hundred sixty-nine thousand, two hundred seventy-five.

   e. 97,972,266
   97 million, 972 thousand, 266
   Ninety-seven million, nine hundred seventy-two thousand, two hundred sixty-six.

Exercise #3, page 13

1. a. (i) millions
   (ii) units
   (iii) 74,000,091; thousands; hundreds

   b. (i) 6
   (ii) millions
   (iii) 4, last
   (iv) 319
   (v) 6,076,004,319; hundred millions; hundred thousands; ten thousands

2. a. 53,092,014
   b. 70,004,900,012
   c. 4,002,000
   d. 15,040,600
   e. 300,000,906

End of Lesson 1

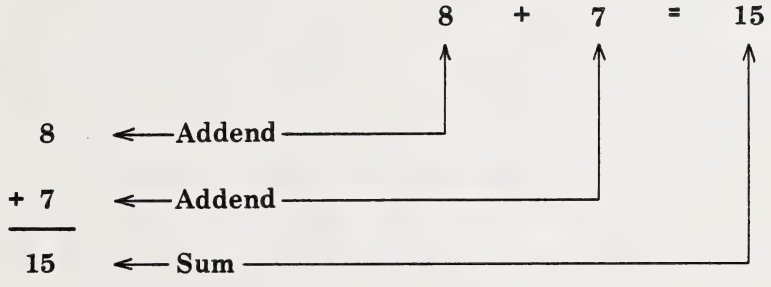
Practical Mathematics
Module 1
Lesson 2

ADDITION AND SUBTRACTION OF WHOLE NUMBERS

Topic One: Addition

Addition is the operation we use to find the sum of two or more numbers without counting.

In addition, the numbers that are added are called the ADDENDS and the result that is obtained is called the SUM. The symbol + is used to represent the operation of addition.



A. Basic Addition Facts

The ten digits of our numeration system 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, if added together two at a time, make 100 combinations. It is these 100 combinations that must be practised until they can be recognized automatically.

The chart on page 2 gives the 100 basic combinations. If you are unsure of any of the facts, use the chart as reference.

To find an addition combination on the chart, such as 5 + 3, find 5 in the column on the left. With your finger trace along the row containing 5 until you come to the column which has 3 at the top. The sum of 5 + 3, or 8, is in that square.

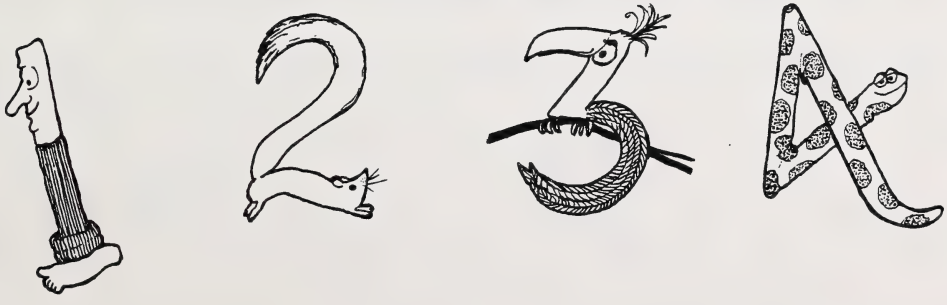


Table of Basic Addition Facts

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Self-Correcting Exercise #1

Find the sum of each combination given below. Use the chart to check your answers.

a. $\begin{array}{r} 4 \\ +\ 7 \\ \hline \end{array}$ b. $\begin{array}{r} 8 \\ +\ 5 \\ \hline \end{array}$ c. $\begin{array}{r} 9 \\ +\ 8 \\ \hline \end{array}$ d. $\begin{array}{r} 7 \\ +\ 6 \\ \hline \end{array}$

e. $\begin{array}{r} 9 \\ +\ 3 \\ \hline \end{array}$ f. $\begin{array}{r} 0 \\ +\ 4 \\ \hline \end{array}$ g. $\begin{array}{r} 6 \\ +\ 2 \\ \hline \end{array}$ h. $\begin{array}{r} 1 \\ +\ 8 \\ \hline \end{array}$

EXERCISE - BASIC ADDITION FACTS

1. You should be able to give the sums of the following basic facts quickly. Try to get them all correct.

a. $\begin{array}{r} 4 \\ +\ 6 \\ \hline \end{array}$	b. $\begin{array}{r} 9 \\ +\ 4 \\ \hline \end{array}$	c. $\begin{array}{r} 5 \\ +\ 9 \\ \hline \end{array}$	d. $\begin{array}{r} 6 \\ +\ 3 \\ \hline \end{array}$
e. $\begin{array}{r} 7 \\ +\ 9 \\ \hline \end{array}$	f. $\begin{array}{r} 6 \\ +\ 8 \\ \hline \end{array}$	g. $\begin{array}{r} 9 \\ +\ 8 \\ \hline \end{array}$	h. $\begin{array}{r} 7 \\ +\ 5 \\ \hline \end{array}$
i. $\begin{array}{r} 1 \\ +\ 0 \\ \hline \end{array}$	j. $\begin{array}{r} 8 \\ +\ 7 \\ \hline \end{array}$	k. $\begin{array}{r} 9 \\ +\ 7 \\ \hline \end{array}$	l. $\begin{array}{r} 6 \\ +\ 9 \\ \hline \end{array}$
m. $\begin{array}{r} 8 \\ +\ 9 \\ \hline \end{array}$	n. $\begin{array}{r} 9 \\ +\ 3 \\ \hline \end{array}$	o. $\begin{array}{r} 6 \\ +\ 7 \\ \hline \end{array}$	p. $\begin{array}{r} 0 \\ +\ 0 \\ \hline \end{array}$
q. $\begin{array}{r} 2 \\ +\ 5 \\ \hline \end{array}$	r. $\begin{array}{r} 4 \\ +\ 5 \\ \hline \end{array}$	s. $\begin{array}{r} 7 \\ +\ 4 \\ \hline \end{array}$	t. $\begin{array}{r} 8 \\ +\ 4 \\ \hline \end{array}$
u. $\begin{array}{r} 7 \\ +\ 0 \\ \hline \end{array}$	v. $\begin{array}{r} 0 \\ +\ 4 \\ \hline \end{array}$	w. $\begin{array}{r} 9 \\ +\ 0 \\ \hline \end{array}$	x. $\begin{array}{r} 3 \\ +\ 0 \\ \hline \end{array}$

2. Tell whether the arrow points to an addend or a sum. The first one is done for you.

a. $\begin{array}{r} 7 \\ +\ 6 \\ \hline 13 \end{array}$ ← *addend*

b. $9 + 3 = 12$ ↗ _____

c. $\begin{array}{r} 7 \\ +\ 2 \\ \hline 9 \end{array}$ ← _____

d. $\begin{array}{r} 0 \\ +\ 1 \\ \hline 1 \end{array}$ ← _____ e. $3 + 2 = 5$ ↖ _____ f. $8 + 0 = 8$ ↗ _____

3. Write the missing number or word in each sentence.

a. In the number sentence 7 + 9 = 16, the number 16 is the _____ of the numbers 7 and 9.

b. In the number sentence 3 + 4 = 7, the numbers _____ and _____ are addends whose _____ is 7.

c. In the number sentence 7 + 2 = 9, 7 and 2 are _____ and the number 9 is the _____.

B. Zero As An Addend

Find the following sums.

a. $\begin{array}{r} 3 \\ +\ 0 \\ \hline \end{array}$ b. $\begin{array}{r} 0 \\ +\ 9 \\ \hline \end{array}$ c. $\begin{array}{r} 7 \\ +\ 0 \\ \hline \end{array}$ d. $\begin{array}{r} 6 \\ +\ 0 \\ \hline \end{array}$

e. 5 + 0 = _____ f. 0 + 7 = _____ g. 2 + 0 = _____

What was the common addend in each exercise? _____

Did you notice any relationship between the remaining addend and the sum?

_____

_____

It does not matter what number is added to zero, the number remains unchanged. Thus addition involving zero is straightforward.

In general, $\begin{array}{r} a \\ +\ 0 \\ \hline a \end{array}$ $a + 0 = a$

The sum of 0 and any number is that number.

C. The Commutative Property - Changing the Order of the Addends

$7 + 5 = 12$

$5 + 7 = 12$

$7 + 5 = 5 + 7$

$$\begin{array}{rcl} 7 & \longleftarrow \text{addend} \longrightarrow & 5 \\ +\ 5 & \longleftarrow \text{addend} \longrightarrow & +\ 7 \\ 12 & \longleftarrow \text{sum} \longrightarrow & 12 \end{array}$$

The order in which we add two numbers does not affect the sum.

EXERCISE - COMMUTATIVE PROPERTY OF ADDITION

Fill in the blanks. You should not have to do any computation.

1. $25 + 7 =$ _____ $+ 25$
2. $37 + 59 =$ _____ $+ 37$
3. $369 +$ _____ $= 923 + 369$
4. $1768 + 4219 = 4219 +$ _____
5. $219 + 778 =$ _____ $+ 219$
6. $145,507 +$ _____ $= 11,789 + 145,507$

D. The Associative Property - Changing the Grouping of Addends

We can add more than two numbers together to obtain a sum. However only two may be combined at any one time. We would add two addends and obtain a sum; this sum would then become an addend and we would combine it with another addend to obtain another sum. This process can continue until we have combined all the addends.

When adding three numbers, we have two choices of which addends we can group together and add first. (This is done without changing the order in which the addends appear.)

We can change the grouping of numbers and obtain the same sum.

EXAMPLE:

$$5 + 6 + 9 \qquad\qquad 5 + 6 + 9$$
$$= (5 + 6) + 9 \qquad\qquad = 5 + (6 + 9)$$
$$= 11 + 9 \qquad\qquad = 5 + 15$$
$$= 20 \qquad\qquad = 20$$

We can first take 5 + 6, and then add 9, or we can add 5 to the sum of 6 and 9. Note that the order of the numbers does not change but the brackets are changed.

EXAMPLE: Use the associative property to find the sum 5 + 4 + 7 in two different ways.

Solution

Grouping the 5 and 4 together first, we get:
$5 + 4 + 7 = (5 + 4) + 7 = 9 + 7 = 16$
Grouping the 4 and 7 together first, we get:
$5 + 4 + 7 = 5 + (4 + 7) = 5 + 11 = 16$

The associative property can be very useful. Some of the addition combinations are easier than others. It is easier to add when we can form a grouping that gives a sum which ends in zero.

EXAMPLE: $5 + 7 + 3 = \blacksquare$

Solution

We could choose to add (5 + 7) first, but if we group the question 5 + (7 + 3), we have a grouping whose sum ends in 0.

$$5 + 7 + 3 = 5 + (7 + 3)$$
$$= 5 + 10 \leftarrow \textit{Sum ends in zero}$$
$$= 15$$

EXAMPLE: Add: 6 + 4 + 9

Solution

$$6 + 4 + 9 = (6 + 4) + 9$$
$$= 10 + 9 \leftarrow \textit{Sum ends in zero}$$
$$= 19$$

EXERCISE - ASSOCIATIVE PROPERTY OF ADDITION

1. Show by means of the associative property that we can change the grouping of the addends to obtain the same sum. The first one is done for you.

a. $7 + 4 + 6$

① $7+4+6 = (7+4)+6$
$= 11+6$
$= 17$

② $7+4+6 = 7+(4+6)$
$= 7+10$
$= 17$

$\therefore (7+4)+6 = 7+(4+6)$

↖ This symbol means "therefore."

b. b. $1 + 9 + 8$

c. $7 + 8 + 2$

d. $3 + 5 + 5$

2. Find each sum. Choose the grouping that is most helpful. Do not change the order. Show which addends you have grouped first. The first question has been done for you.

a. $\begin{matrix} 3 \\ 7 \end{matrix}\Big\}$ group first
$\underline{+\ 5}$
15

b. $2 + 8 + 3 =$ _____

c. $5 + 5 + 8 =$ _____

d. $\begin{array}{r} 8 \\ 2 \\ +\ 7 \end{array}$ e. $\begin{array}{r} 4 \\ 6 \\ +\ 9 \end{array}$ f. $\begin{array}{r} 5 \\ 5 \\ +\ 1 \end{array}$

E. Rearranging Addends

Think about the three addends in the sum 3 + 4 + 8. By the commutative property we know that we can change the order of the addends and obtain the same sum.

By the associative property we know that we can group the addends differently and obtain the same sum. We can use these two properties together and begin by adding any two numbers in any order.

Fill in the blanks in the chart below.

$\overset{\swarrow\ \searrow}{3 + 4 + 8}$	$\overset{\swarrow\ \searrow}{3 + 4 + 8}$	$\overset{\downarrow\ \quad \downarrow}{3 + 4 + 8}$
(Brackets below indicate that numbers 3 and 4 are added first.)	(Numbers 4 and 8 are added first.)	(Numbers 3 and 8 are added first.)
$(3 + 4) + 8 = \underline{7} + \underline{8}$ $= \underline{15}$	$3 + (4 + 8) = \underline{\quad} + \underline{12}$ $= \underline{\quad}$	$(3 + 8) + 4 = \underline{11} + \underline{\quad}$ $= \underline{\quad}$
$(4 + 3) + 8 = \underline{\quad} + \underline{\quad}$ $= \underline{\quad}$	$3 + (8 + 4) = \underline{\quad} + \underline{\quad}$ $= \underline{\quad}$	$(8 + 3) + 4 = \underline{\quad} + \underline{\quad}$ $= \underline{\quad}$
$8 + (3 + 4) = \underline{\quad} + \underline{\quad}$ $= \underline{\quad}$	$(4 + 8) + 3 = \underline{\quad} + \underline{\quad}$ $= \underline{\quad}$	$4 + (3 + 8) = \underline{\quad} + \underline{\quad}$ $= \underline{\quad}$
$8 + (4 + 3) = \underline{\quad} + \underline{\quad}$ $= \underline{\quad}$	$(8 + 4) + 3 = \underline{\quad} + \underline{\quad}$ $= \underline{\quad}$	$4 + (8 + 3) = \underline{\quad} + \underline{\quad}$ $= \underline{\quad}$

Can you see where the commutative and associative properties have been used in the steps above?

When we use the commutative and associative properties together to find sums we can arrange and group the addends in any way and still obtain the same sum.

EXERCISE - Adding One-Digit Numbers

1. Fill in the blanks.

   a. The commutative property of addition tells us that we can change the _____ of addends without affecting the sum. Hence,

   $$6 + 4 = 4 + __$$
   $$8 + __ = 6 + 8$$

   b. The associative property of addition tells us that we can change the manner of _____ addends without affecting the sum. Hence,

   $$(9 + 3) + 7 = 9 + (__ + 7)$$
   $$4 + (1 + __) = (4 + 1) + 5$$

   c. The _____ and _____ properties combined allow us to arrange and group addends in any way and still obtain the same sum. Hence,

   $$6 + (8 + 4) = (6 + __) + 8$$
   $$(4 + 2) + (6 + 8) = (4 + __) + (2 + 8)$$

2. Find the sums.

a. $\begin{array}{r} 8 \\ 1 \\ +\ 5 \\ \hline \end{array}$ b. $\begin{array}{r} 2 \\ 9 \\ +\ 7 \\ \hline \end{array}$ c. $\begin{array}{r} 3 \\ 5 \\ +\ 0 \\ \hline \end{array}$ d. $\begin{array}{r} 9 \\ 1 \\ +\ 8 \\ \hline \end{array}$

e. $\begin{array}{r} 8 \\ 2 \\ 7 \\ +\ 5 \\ \hline \end{array}$ f. $\begin{array}{r} 7 \\ 1 \\ 8 \\ +\ 2 \\ \hline \end{array}$ g. $\begin{array}{r} 6 \\ 4 \\ 7 \\ 8 \\ +\ 2 \\ \hline \end{array}$ h. $\begin{array}{r} 9 \\ 7 \\ 3 \\ 2 \\ 5 \\ +\ 2 \\ \hline \end{array}$

i. $\begin{array}{r} 7 \\ 9 \\ 8 \\ +\ 5 \\ \hline \end{array}$ j. $6 + 2 + 3 + 5 =$ _____ k. $8 + 4 + 0 + 6 + 2 =$ _____

F. Checking Addition

When we are finding sums we want our work to be as accurate as possible. A simple way to check an addition is to add the same numbers in the opposite order. We can use this method because the commutative property tells us that the order in which a set of numbers is added does not affect the sum of the numbers.

EXAMPLE: Add.

$$\begin{array}{r} 8 \\ 7 \\ 6 \\ +\ 4 \\ \hline 25 \end{array}$$

Think: "8 and 7 is 15
15 and 6 is 21
21 and 4 is 25"

Check.

$$\begin{array}{r} 25 \\ \hline 8 \\ 7 \\ 6 \\ 4 \end{array}$$

Start at the bottom.
Think: "4 and 6 is 10
10 and 7 is 17
17 and 8 is 25"

EXERCISE - CHECKING ADDITION

Find each sum. Check your work by adding in the opposite order. The first one has been done for you.

1.

$$\begin{array}{r} 4 \\ 7 \\ 8 \\ +\ 3 \\ \hline 22 \end{array}$$

Check:

$$\begin{array}{r} 22 \\ \hline 4 \\ 7 \\ 8 \\ +\ 3 \end{array}$$

2.

$$\begin{array}{r} 5 \\ 8 \\ 2 \\ +\ 6 \\ \hline \end{array}$$

Check:

3.

$$\begin{array}{r} 8 \\ 1 \\ 7 \\ +\ 9 \\ \hline \end{array}$$

Check:

4.

$$\begin{array}{r} 2 \\ 7 \\ 7 \\ 1 \\ 5 \\ +\ 6 \\ \hline \end{array}$$

Check:

G. Addition of a Two-Digit Number and a One-Digit Number

Speed and accuracy are two prime objectives in mathematics. When adding figures you must be able to do so as quickly as possible, and your answers must be correct.

To gain speed when adding columns of numbers you must be able to give on sight the sum of any two-figure and any one-figure number.

EXAMPLE: Add 65 and 8.

Solution

```
  65
+ 8
  73
```

You should be able to obtain the sum 73 in one step. You should not have to think "5 and 8 are 13, carry 1; 6 and 1 are 7; the sum is 73."

EXERCISE - ADDITION OF TWO-DIGITS TO ONE DIGIT

Try to add these on sight without having to carry from one column to the next.

1. 37 + 5	2. 27 + 9	3. 20 + 9	4. 28 + 8
5. 26 + 7	6. 57 + 9	7. 46 + 9	8. 93 + 4
9. 26 + 9	10. 19 + 4	11. 55 + 8	12. 39 + 7

Now add the following columns. Try to work quickly and accurately.

13.
$$\begin{array}{r} 2 \\ 7 \\ 8 \\ 3 \\ 9 \\ +\ 1 \end{array}$$

14.
$$\begin{array}{r} 4 \\ 2 \\ 7 \\ 2 \\ 5 \\ 4 \\ 1 \\ 8 \\ 3 \\ +\ 8 \end{array}$$

15.
$$\begin{array}{r} 4 \\ 8 \\ 3 \\ 4 \\ 2 \\ 2 \\ 7 \\ 8 \\ +\ 7 \end{array}$$

16.
$$\begin{array}{r} 1 \\ 6 \\ 3 \\ 4 \\ 9 \\ 7 \\ 5 \\ 4 \\ 4 \\ +\ 1 \end{array}$$

H. Addition of Numbers with Two Digits

Now that you have gained skill at addition of numbers with one digit, let's move on to addition of numbers with two-digits.

EXAMPLE: Find the sum of 74 and 19.

Solution

$$\begin{array}{r} {}^{1}7\,4 \\ +\ 1\,9 \\ \hline 3 \end{array}$$

Think: 4 + 9 is 13. 13 can be regrouped as 1 ten and 3 ones. Write the 3 under the ones column and carry the 1 ten to the tens column.

$$\begin{array}{r} {}^{1}7\,4 \\ +\ 1\,9 \\ \hline 9\,3 \end{array}$$

Think: 1 + 7 is 8 and 8 + 1 is 9. Write the 9 under the tens column.

When adding two-digit numbers it is important to work carefully when you carry a digit from one column to another.

Self-Correcting Exercise #2

Find the following sums. The answers are found on page 25 of this lesson. Correct your own answers and be sure to re-do any questions you had incorrect.

	a	b	c	d	e	f	g
1.	21 + 34	19 + 45	24 + 77	28 + 49	47 + 23	29 + 63	79 + 20
2.	24 + 39	72 + 35	29 + 45	41 + 38	25 + 39	68 + 24	47 + 32
3.	25 + 42	28 + 24	46 + 37	28 + 45	66 + 12	59 + 26	32 + 19
4.	84 + 14	48 + 83	11 + 88	79 + 20	29 + 63	47 + 23	28 + 49

EXERCISE - ADDING TWO-DIGIT NUMBERS

1.	94 + 82	2. 32 + 74	3. 36 + 92	4. 19 + 79
5.	95 + 62	6. 21 + 76	7. 48 + 25	9. 83 + 49

I. Addition of Numbers with More Than Two Digits

EXAMPLE: Add 1749 and 598.

Solution

```
  1 7 4 9
+   5 9 8
      7
```

Think: "9 and 8 is 17." Write the 7 under the ones column and carry the 1 to the tens column

```
  1 7 4 9
+   5 9 8
     47
```

Think: "1 and 4 is 5, 5 and 9 is 14." Write the 4 under the tens column and carry the 1 to the hundreds column.

```
  1 7 4 9
+   5 9 8
    347
```

Think: "1 and 7 is 8. 8 and 5 is 13". Write the 3 under the hundreds column and carry the 1 to the thousands column.

```
  1 7 4 9
+   5 9 8
   2347
```

Think: "1 and 1 is 2". Write 2 under the thousands column.

1749 + 598 = 2347

Self-Correcting Exercise #3

Find the following sums. The answers are found on page 25 of this lesson. Correct your own answers and be sure to re-do any questions you had incorrect.

1. $\begin{array}{r} 6275 \\ +\ 469 \\ \hline \end{array}$

2. $\begin{array}{r} 6073 \\ +\ 1874 \\ \hline \end{array}$

3. $\begin{array}{r} 7974 \\ +\ 3851 \\ \hline \end{array}$

4. $\begin{array}{r} 5376 \\ 9852 \\ +\ 1384 \\ \hline \end{array}$

5. $\begin{array}{r} 81 \\ 3277 \\ 778 \\ +\ 810 \\ \hline \end{array}$

6. $\begin{array}{r} 392 \\ 40 \\ 90 \\ +\ 1773 \\ \hline \end{array}$

7. $\begin{array}{r} 9527 \\ 389 \\ 5320 \\ +\ 607 \\ \hline \end{array}$

8. $\begin{array}{r} 898 \\ 781 \\ 2 \\ +\ 505 \\ \hline \end{array}$

EXERCISE - ADDING NUMBERS WITH MORE THAN TWO DIGITS

Add. Show the numbers you have carried over, as in the solution to #1.

1.
$$\begin{array}{r} {}^{1}9{}^{1}25 \\ {}^{3}1572 \\ 931 \\ +\ 716 \\ \hline 4144 \end{array}$$

2.
$$\begin{array}{r} 856 \\ 1372 \\ 684 \\ 923 \\ +\ 48 \end{array}$$

3.
$$\begin{array}{r} 1973 \\ 832 \\ 961 \\ +\ 29 \end{array}$$

4.
$$\begin{array}{r} 568 \\ 4173 \\ 92 \\ 624 \\ +\ 48 \end{array}$$

5.
$$\begin{array}{r} 236 \\ 574 \\ 28 \\ 169 \\ +\ 64 \end{array}$$

6.
$$\begin{array}{r} 657 \\ 790 \\ 94 \\ 412 \\ +\ 67 \end{array}$$

7.
$$\begin{array}{r} 87 \\ 562 \\ 394 \\ 76 \\ +\ 84 \end{array}$$

8.
$$\begin{array}{r} 91 \\ 572 \\ 618 \\ 204 \\ +\ 67 \end{array}$$

9.
$$\begin{array}{r} 6732 \\ 486 \\ 674 \\ +\ 218 \end{array}$$

10.
$$\begin{array}{r} 1743 \\ 685 \\ 21 \\ 605 \\ +\ 69 \end{array}$$

11.
$$\begin{array}{r} 1846 \\ 286 \\ 9571 \\ 841 \\ +\ 69 \end{array}$$

12.
$$\begin{array}{r} 8988 \\ 736 \\ 971 \\ 1572 \\ +\ 208 \end{array}$$

13.
$$\begin{array}{r} 428 \\ 9 \\ 35723 \\ 78 \\ +\ 531 \end{array}$$

14.
$$\begin{array}{r} 49 \\ 6973 \\ 682 \\ 91 \\ +\ 40839 \end{array}$$

15.
$$\begin{array}{r} 7 \\ 8281 \\ 571 \\ 59 \\ +\ 8 \end{array}$$

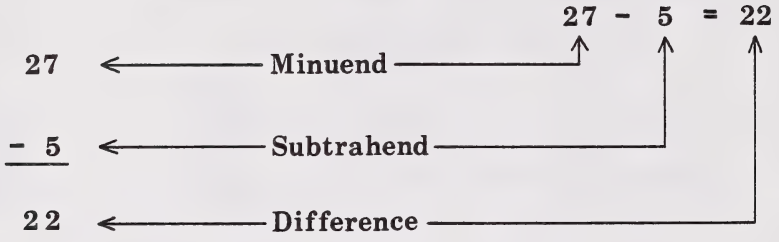
16.
$$\begin{array}{r} 736 \\ 32 \\ 8177 \\ 8 \\ +\ 317 \end{array}$$

Topic Two: Subtraction

Subtraction is the operation that we use to find the amount that remains when one number is taken from another. Subtraction is the reverse of adding two numbers. In addition, we seek a sum of two given addends, while in subtraction we seek one of the addends of a given sum.

In subtraction, the number from which we subtract is called the MINUEND. The number which is subtracted is called the SUBTRAHEND. The result that is obtained is called the DIFFERENCE. The symbol - is used to represent the operation of subtraction.

EXAMPLE:



Fill in the blanks.

1. In the number sentence 15 - 2 = 13, the number 2 is the _____ and the number 15 is the _____. The difference is _____.

2. What is the difference when the minuend is 10 and the subtrahend is 7? _____

3. What is the symbol that indicates the operation of subtraction? _____

4. In the number sentence 16 - 2 = 14, the number 16 is the _____; the subtrahend is _____ and the difference is _____.

A. Subtraction Facts

Just as in addition, there are basic combinations in subtraction that we must practise until they can be recognized automatically without counting or guessing.

The same chart on page 2 that was used for the basic addition facts can also be used to study the subtraction combinations.

EXAMPLE: Refer to the chart on page 2.
To find a subtraction combination such as 12 - 3 = ■ find 3 in the column at the left. With your finger trace along the row containing 3 until you come to 12. The number at the top of this column is the answer to the fact 12 - 3, or 9.

Self-Correcting Exercise #4

See how well you do in subtraction of the basic facts. If you have difficulty, refer back to the chart on page 2 to help you learn the facts you do not know. The answers are found on page 25 of this lesson. Check your own work and correct any errors you made.

1. 11 - 9	2. 13 - 9	3. 5 - 1	4. 11 - 5	5. 14 - 8
6. 10 - 2	7. 14 - 6	8. 14 - 9	9. 11 - 4	10. 8 - 1
11. 13 - 8	12. 13 - 7	13. 9 - 3	14. 14 - 5	15. 12 - 5
16. 8 - 4	17. 6 - 6	18. 9 - 0	19. 9 - 9	20. 12 - 4
21. 11 - 8	22. 7 - 5	23. 6 - 4	24. 15 - 9	25. 17 - 8

B. Subtraction Without Renaming

The basic subtraction facts can now be used to do other subtraction exercises.

EXAMPLE: Subtract 7 from 49.

Solution

$$\begin{array}{r} 49 \\ -\ 7 \\ \hline 2 \end{array}$$

Think: "9 - 7 is 2". Write the 2 in the ones column.

$$\begin{array}{r} 49 \\ -\ 7 \\ \hline 42 \end{array}$$

Think: "There is nothing to subtract from 4, or 4 - 0 is 4". Write 4 in the tens column.

EXAMPLE: Subtract 23 from 78.

Solution

Step 1

$$\begin{array}{r} 78 \\ -\ 23 \\ \hline 5 \end{array}$$

Step 2

$$\begin{array}{r} 78 \\ -\ 23 \\ \hline 55 \end{array}$$

We always begin subtracting the numbers in the farthest right column and move to the left.

EXERCISE - SUBTRACTION WITHOUT RENAMING

Find the difference.

1. $\begin{array}{r} 16 \\ -\ 9 \end{array}$	2. $\begin{array}{r} 49 \\ -\ 4 \end{array}$	3. $\begin{array}{r} 85 \\ -\ 3 \end{array}$	4. $\begin{array}{r} 98 \\ -67 \end{array}$	5. $\begin{array}{r} 99 \\ -89 \end{array}$
6. $\begin{array}{r} 182 \\ -\ 81 \end{array}$	7. $\begin{array}{r} 358 \\ -246 \end{array}$	8. $\begin{array}{r} 59 \\ -\ 8 \end{array}$	9. $\begin{array}{r} 72 \\ -61 \end{array}$	10. $\begin{array}{r} 49 \\ -23 \end{array}$
11. $\begin{array}{r} 64 \\ -13 \end{array}$	12. $\begin{array}{r} 49 \\ -23 \end{array}$	13. $\begin{array}{r} 53 \\ -31 \end{array}$	14. $\begin{array}{r} 65 \\ -14 \end{array}$	15. $\begin{array}{r} 29 \\ -17 \end{array}$

C. Renaming In Subtraction

Often we can not subtract directly.

EXAMPLE: Subtract 9 from 43.

Solution

$$\begin{array}{r} 43 \\ -\ 9 \end{array}$$

When we try to subtract 9 from 43, we discover that we are unable to subtract 9 from 3. Therefore we RENAME the 43 as 3 TENS + 13 ONES.

$$\begin{array}{rcl} 43 & = & 4 \text{ TENS} + 3 \text{ ONES} \\ -\ 9 & = & \underline{\qquad\qquad - 9 \text{ ONES}} \end{array} \quad = \quad \begin{array}{r} 3 \text{ TENS} + 13 \text{ ONES} \\ \underline{\qquad\qquad - \ 9 \text{ ONES}} \\ 3 \text{ TENS} + 4 \text{ ONES} \end{array} = 34$$

Notice that 43 was renamed as 3 tens + 13 ones so that you could subtract.

The shorter methods may be used to save time once you understand the procedure.

$$\begin{array}{r} {}^{3}\cancel{4}\,{}^{13}\cancel{3} \\ -\ 9 \\ \hline 34 \end{array} \qquad \text{OR} \qquad \begin{array}{r} {}^{3}\cancel{4}\,{}^{1}3 \\ -\ 9 \\ \hline 34 \end{array}$$

Self-Correcting Exercise #5

To give you practise renaming in subtraction work these questions. The answers are found on page 25 of this lesson. Check your own work and correct any errors you made.

1. $\begin{array}{r} 62 \\ -38 \end{array}$	2. $\begin{array}{r} 65 \\ -36 \end{array}$	3. $\begin{array}{r} 71 \\ -43 \end{array}$	4. $\begin{array}{r} 52 \\ -18 \end{array}$
5. $\begin{array}{r} 91 \\ -46 \end{array}$	6. $\begin{array}{r} 83 \\ -29 \end{array}$	7. $\begin{array}{r} 41 \\ -27 \end{array}$	8. $\begin{array}{r} 85 \\ -38 \end{array}$
9. $\begin{array}{r} 63 \\ -34 \end{array}$	10. $\begin{array}{r} 52 \\ -26 \end{array}$	11. $\begin{array}{r} 82 \\ -47 \end{array}$	12. $\begin{array}{r} 70 \\ -34 \end{array}$

EXERCISE - RENAMING IN SUBTRACTION

Find the difference.

1. 54 - 8	2. 95 - 6	3. 81 - 7	4. 65 - 9	5. 45 - 7
6. 31 - 5	7. 53 - 4	8. 33 - 7	9. 22 - 9	10. 54 - 5
11. 90 - 2	12. 97 - 9	13. 23 -19	14. 88 -79	15. 29 -17

D. Renaming More Than Once In Subtraction

You have seen that it is sometimes necessary to rename the larger number before you subtract. Often you have to rename more than once to complete the subtraction.

EXAMPLE: Subtract 179 from 263.

Solution

$$\begin{array}{r} 263 \\ -179 \end{array} = \begin{array}{r} 200 + 60 + 3 \\ -100 + 70 + 9 \end{array} = \begin{array}{r} 200 + (50 + 13) \\ -100 + 70 + 9 \\ \hline ? \quad 4 \end{array} = \begin{array}{r} (100 + 150) + 13 \\ -100 + 70 + 9 \\ \hline 0 + 80 + 4 \end{array} = 84$$

60 was renamed 50 + 13 *250 was renamed 100 + 150*

The shorter methods can be used once you understand the procedure.

$$\begin{array}{r} 263 \\ -179 \\ \hline \end{array}$$

We can't subtract 9 from 3, so we regroup 6 tens and 3 ones as 5 tens and 13 ones.

$$\begin{array}{r} 2\,\overset{5}{\cancel{6}}\,\overset{13}{3} \\ -179 \\ \hline 4 \end{array}$$

Put a stroke through the 6 and replace it with a 5. Add a 1 in front of the 3 to make 13 ones. 13 - 9 = 4

$$\begin{array}{r} \overset{1}{2}\,\overset{5}{\cancel{6}}\,\overset{13}{3} \\ -179 \\ \hline 4 \end{array}$$

We can't subtract 7 from 5, so we regroup 2 hundreds and 5 tens as 1 hundred and 15 tens.

$$\begin{array}{r} \overset{1}{\cancel{2}}\,\overset{15}{\cancel{6}}\,\overset{13}{3} \\ -179 \\ \hline 84 \end{array}$$

Put a stroke through the 2 and replace it with a 1. Add a 1 in front of the 5 to make 15 tens. 15 - 7 = 8

263 - 179 = 84

Special thought must be given when the minuend involves one or more zeros.

EXAMPLE: Subtract 298 from 800.

Solution

$$\begin{array}{r} 800 \\ -298 \\ \hline \end{array} = \begin{array}{r} 800 + 00 + 0 \\ -200 + 90 + 8 \\ \hline \end{array} = \begin{array}{r} \overbrace{700 + 100}^{\text{800 was renamed as } (700 + 100)} + 0 \\ -200 + 90 + 8 \\ \hline \end{array} = \begin{array}{r} 700 + \overbrace{90 + 10}^{\text{100 was renamed as } (90 + 10)} \\ -200 + 90 + 8 \\ \hline 500 + 00 + 2 \end{array} = 502$$

The long method above illustrates the ideas we must think about through each step of the subtraction.

In your work you will probably use the shorter method.

$$\begin{array}{r} \overset{7}{\cancel{8}}\,\overset{9}{\cancel{0}}\,\overset{10}{0} \\ -298 \\ \hline 502 \end{array} \quad \text{OR} \quad \begin{array}{r} \overset{7}{\cancel{8}}\,\overset{9}{\cancel{0}}\,0 \\ -298 \\ \hline 502 \end{array}$$

First, the 8 is crossed out and changed to a 7, and the middle digit becomes 10. Next, the 10 is crossed out and changed to a 9, and the last digit becomes 10. Then you subtract.

EXERCISE - RENAMING MORE THAN ONCE IN SUBTRACTION

Subtract. Show borrowing as in the solution to #1.

1. $3\overset{5}{\cancel{9}}\overset{1}{2}$ $-\ 48$ 344

2. 204 $-\ 69$

3. 144 $-\ 85$

4. 206 $-\ 39$

5. 137 $-\ 48$

6. 433 $-\ 24$

7. 182 $-\ 98$

8. 169 $-\ 89$

9. 104 $-\ 75$

10. 114 $-\ 37$

11. 241 $-\ 82$

12. 155 $-\ 66$

13. 1000 $-\ 852$

14. 1509 $-\ 868$

15. 9070 $-\ 598$

16. 9317 -4432

17. 9361 -4327

18. 8802 -1326

19. 7456 -4679

20. 8510 -5889

E. Checking Subtraction

We have said that subtraction is the operation of finding another addend when the sum and one addend are known. Subtraction is the reverse operation of addition. This relationship can be used to check subtraction.

EXAMPLE:

$$\begin{array}{r} {}^{1}7{}^{1}2{}^{1}4 \\ -\,3\,0\,9 \\ \hline 4\,1\,5 \end{array} \qquad \text{Check:} \qquad \begin{array}{r} 4\,1\,5 \\ +\,3\,0\,9 \\ \hline 7\,2\,4 \end{array}$$

The check can also be performed without rewriting the numbers. The sum of the difference and the subtrahend is the same as the minuend.

$$\begin{array}{r} {}^{1}7{}^{1}2{}^{1}4 \\ -\,3\,0\,9 \\ \hline 4\,1\,5 \\ \hline 7\,2\,4 \end{array}$$

To check, add the difference and the subtrahend. If your sum is the same as the minuend, your subtraction is correct.

EXERCISE - CHECKING SUBTRACTION

Subtract. Check each answer. Show your work.

1. $\begin{array}{r} {}^{3}\cancel{4}{}^{1}7 \\ -\,3\,8 \\ \hline 9 \end{array}$ Check: $\begin{array}{r} 9 \\ +\,3\,8 \\ \hline 4\,7 \end{array}$

2. $\begin{array}{r} 6\,3 \\ -\,3\,9 \\ \hline \end{array}$ Check:

Check the following without rewriting the numbers.

3. $\begin{array}{r} 2\,0\,6 \\ -\,1\,2\,7 \\ \hline \end{array}$

4. $\begin{array}{r} 2\,0\,0\,0 \\ -\,6\,6\,2 \\ \hline \end{array}$

5. $\begin{array}{r} 1\,2\,3\,1 \\ -\,4\,1\,1 \\ \hline \end{array}$

6. $\begin{array}{r} 3\,0\,0\,3 \\ -\,4\,2\,6 \\ \hline \end{array}$

EXERCISE - ADDITION AND SUBTRACTION OF WHOLE NUMBERS

Work carefully. Remember to watch for the addition and subtraction symbols. In the spaces show all your computation.

1. $2553 - 695 =$ _____

2. $7143 + 998 =$ _____

3. $8927 - 549 =$ _____

4. $3295 - 576 =$ _____

5. $7276 + 465 - 826 =$ _____

6.
$$\begin{array}{r} 6529 \\ 762 \\ 407 \\ 8123 \\ +\ 49 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 10000 \\ -\ 9278 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 7 \\ 2 \\ 3 \\ 8 \\ +\ 8 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 437 \\ 925 \\ 819 \\ +\ 744 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 779 \\ 452 \\ 840 \\ +\ 109 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 7003 \\ -\ 94 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 1977 \\ -\ 869 \\ \hline \end{array}$$

13.
$$\begin{array}{r} 2824 \\ 143 \\ 14 \\ 309 \\ +\ 7 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 2220 \\ 314 \\ 77 \\ +\ 891 \\ \hline \end{array}$$

15. Subtract one thousand seven hundred ninety from five thousand eighty-two.

Key To Self-Correcting Exercises in Lesson 2

Exercise #2, page 13

	a	b	c	d	e	f	g
1.	55	64	101	77	70	92	99
2.	63	107	74	79	64	92	79
3.	67	52	83	73	78	85	51
4.	98	131	99	99	92	70	77

Exercise #3, page 14

1. 6744	2. 7947	3. 11825	4. 16612
5. 4946	6. 2295	7. 15843	8. 2186

Exercise #4, page 17

1. 2	2. 4	3. 4	4. 6	5. 6
6. 8	7. 8.	8. 5	9. 7	10. 7
11. 5	12. 6	13. 6	14. 9	15. 7
16. 4	17. 0	18. 9	19. 0	20. 8
21. 3	22. 2	23. 2	24. 6	25. 9

Exercise #5, page 19

1. 24	2. 29	3. 28	4. 34
5. 45	6. 54	7. 14	8. 47
9. 29	10. 26	11. 35	12. 36

End of Lesson 2

Practical Mathematics
Module 1
Lesson 3

MULTIPLICATION OF WHOLE NUMBERS

Multiplication is the operation we use to find the sum of 2 or more addends of the same value without adding.

```
  12
  12
  12
  12
+ 12
  60
```

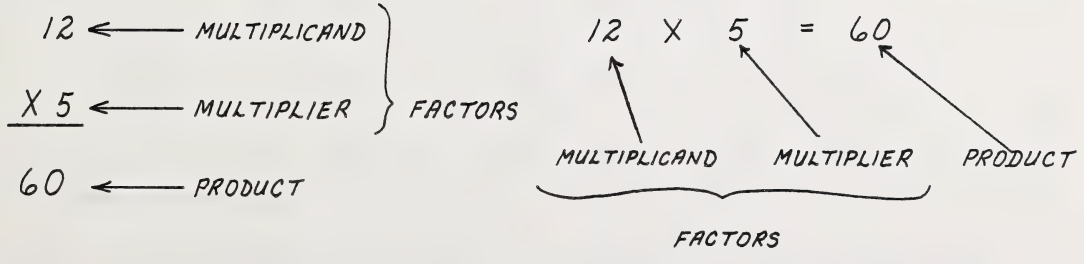
How many times was 12 added?

When we add 12 five times, our sum is 60.

```
  12
× 5
  60
```

Instead of using repeated additions, we can use a shortcut. Multiplication is a shortcut for repeated additions.

In multiplication, the number that you multiply is called the MULTIPLICAND. The number by which you multiply is called the MULTIPLIER. These two numbers are also called the FACTORS. The result that is obtained is called the PRODUCT. The symbol × is used to indicate the operation of multiplication.



A. Basic Multiplication Combinations

The chart below gives the basic combinations of the digits from 1 to 12 and their products. Master this chart thoroughly so that you know the product of any two numbers from 1 to 12 automatically.

BASIC MULTIPLICATION FACTS

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Self-Correcting Exercise #1

How well do you know the following multiplication combinations? The answers will be found on page 24 of this lesson. Check your own work. Redo any questions you had incorrect.

1. 8×7	2. 6×9	3. 0×3	4. 1×1	5. 9×2
6. 7×8	7. 5×3	8. 9×4	9. 9×7	10. 8×8
11. 8×0	12. 6×8	13. 6×7	14. 3×7	15. 5×8

EXERCISE - BASIC MULTIPLICATION FACTS

1. Tell whether the arrow points to the multiplicand, the multiplier, or the product. The first one is done for you.

a. $9 \times 9 = 81$ — *multiplicand*

b. $7 \times 3 = 21$ — _____

c. $9 \times 5 = 45$ — _____

d. $4 \times 0 = 0$ — _____

e. $5 \times 7 = 35$ — _____

f. $0 \times 7 = 0$ — _____

2. Write the missing number or word in each sentence.

a. In the number sentence $3 \times 9 = 27$, the numbers 3 and 9 are _____ and 27 is the _____.

b. In the number sentence $4 \times 6 = 24$, the number 4 is the _____; the factors are _____ and _____; and _____ is the product.

c. In the number sentence $2 \times 9 = 18$, the numbers 2 and 9 are _____. The multiplicand is _____ and the multiplier is _____. The number _____ is the product.

3. Multiply. Try to work as quickly and accurately as possible.

a. 6×0	b. 4×5	c. 4×2	d. 12×12	e. 10×11
f. 2×9	g. 12×7	h. 1×9	i. 0×0	j. 3×9
k. 7×8	l. 4×4	m. 9×7	n. 6×8	o. 7×6
p. 10×12	q. 8×9	r. 7×7	s. 3×9	t. 7×4

B. Zero As A Factor

Find the following products.

a. 1×0	b. 0×7	c. 2×0	d. 12×0	e. 0×10

f. $4 \times 0 =$ _____ g. $9 \times 0 =$ _____ h. $0 \times 5 =$ _____

What was the common factor in each exercise? _____

What was the common product in each exercise? _____

Can you think of a general rule that is shown by the examples on page 4?

_____

When zero is a factor, the product is always zero.

In general, $\begin{array}{r} a \\ \times\ 0 \\ \hline 0 \end{array}$ $0 \times a = 0$

> The product of 0 and any number is 0.

C. One As A Factor

Find the following products.

a. $\begin{array}{r} 1 \\ \times\ 9 \\ \hline \end{array}$ b. $\begin{array}{r} 12 \\ \times\ 1 \\ \hline \end{array}$ c. $\begin{array}{r} 10 \\ \times\ 1 \\ \hline \end{array}$ d. $\begin{array}{r} 1 \\ \times\ 7 \\ \hline \end{array}$ e. $\begin{array}{r} 6 \\ \times\ 1 \\ \hline \end{array}$

f. $1 \times 8 =$ _____ g. $5 \times 1 =$ _____ h. $4 \times 1 =$ _____

What was the common factor in each exercise? _____

Did you notice any relationship between the remaining factor and the product? _____

_____

When a number is multiplied by one, the number remains unchanged.

In general, $\begin{array}{r} a \\ \times\ 1 \\ \hline a \end{array}$ $a \times 1 = a$

> The product of 1 and any number is that number.

D. The Commutative Property - Changing the Order of the Factors

$6 \times 7 = 42$

$7 \times 6 = 42$

$6 \times 7 = 7 \times 6$

$$\begin{array}{rl} 6 & \longleftarrow \text{factor} \longrightarrow 7 \\ \times\ 7 & \longleftarrow \text{factor} \longrightarrow \times\ 6 \\ 42 & \longleftarrow \text{product} \longrightarrow 42 \end{array}$$

The order in which we multiply two numbers does not affect the product.

EXERCISE - COMMUTATIVE PROPERTY OF MULTIPLICATON

Fill in the blanks. You should not have to do any computations.

1. $8 \times 7 =$ _____ $\times 8$
2. $12 \times 10 =$ _____ $\times 12$
3. $11 \times 6 =$ _____ $\times 11$
4. $47 \times 23 =$ _____ $\times 47$
5. $18 \times 40 = 40 \times$ _____
6. $1727 \times 721 = 721 \times$ _____

E. The Associative Property - Changing the Grouping of Factors

In Lesson 2 we learned that addends can be grouped in any way to obtain the same sum. In multiplication, the factors may be grouped in various ways to obtain the same product.

EXAMPLE: $2 \times 3 \times 7 = (2 \times 3) \times 7 = 6 \times 7 = 42$
$2 \times 3 \times 7 = 2 \times (3 \times 7) = 2 \times 21 = 42$

We can first take 2×3, and then multiply by 7, or we can multiply 2 by the product of 3 and 7. Note that the order of the numbers does not change, but the brackets are changed.

We can change the grouping of numbers and obtain the same product.

EXERCISE - ASSOCIATIVE PROPERTY OF MULTIPLICATION

1. Show by means of the associative property that we can change the grouping of the factors to obtain the same product. The first one is done for you.

a. $3 \times 4 \times 6$

$3 \times 4 \times 6 = (3 \times 4) \times 6 = 12 \times 6 = 72$

$3 \times 4 \times 6 = 3 \times (4 \times 6) = 3 \times 24 = 72$

$\therefore (3 \times 4) \times 6 = 3 \times (4 \times 6)$

"therefore"

b. $8 \times 1 \times 9$

c. $10 \times 0 \times 5$
(Watch that zero!)

d. $5 \times 2 \times 11$

e. $12 \times 3 \times 4$

f. $9 \times 8 \times 7$

F. Rearranging Factors

Think about the three factors in the product $3 \times 5 \times 2$. By the commutative property, we know that we can change the order of the factors and obtain the same product. By the associative property, we know that we can group the factors differently and obtain the same product. We can use these two properties together and begin by multiplying any two numbers in any order.

Fill in the blanks in the chart below.

$3 \times 5 \times 2$	$3 \times 5 \times 2$	$3 \times 5 \times 2$
(Brackets below indicate that the numbers 3 and 5 are multiplied first.)	(Numbers 5 and 2 are multiplied first.)	(Numbers 3 and 2 are multiplied first.)
$(3 \times 5) \times 2 = \underline{15} \times \underline{2}$ $= \underline{30}$	$3 \times (5 \times 2) = ___ \times \underline{10}$ $= ___$	$(3 \times 2) \times 5 = \underline{6} \times ___$ $= ___$
$(5 \times 3) \times 2 = ___ \times ___$ $= ___$	$3 \times (2 \times 5) = ___ \times ___$ $= ___$	$(2 \times 3) \times 5 = ___ \times ___$ $= ___$
$2 \times (3 \times 5) = ___ \times ___$ $= ___$	$(5 \times 2) \times 3 = ___ \times ___$ $= ___$	$5 \times (3 \times 2) = ___ \times ___$ $= ___$
$2 \times (5 \times 3) = ___ \times ___$ $= ___$	$(2 \times 5) \times 3 = ___ \times ___$ $= ___$	$5 \times (2 \times 3) = ___ \times ___$ $= ___$

Can you see where the commutative and associative properties have been used in the steps above?

When we use the commutative and associative properties together to find products we can arrange and group the factors in any way and still obtain the same product.

EXERCISE - Multiplying One-Digit Numbers

1. Fill in the blanks.

   a. The commutative property of multiplication tells us that we can change the _____ of factors without affecting the _____. Hence,

   $$3 \times 9 = 9 \times __$$
   $$__ \times 4 = 4 \times 8$$

   b. The associative property of multiplication tells us that we can change the manner of _____ factors without affecting the _____. Hence,

   $$(1 \times 6) \times 9 = 1 \times (6 \times __)$$
   $$4 \times (3 \times __) = (4 \times 3) \times 7$$

   c. The _____ and _____ properties combined allow us to arrange and group factors in any way and still obtain the same product. Hence,

   $$6 \times (5 \times 4) = (5 \times __) \times 4$$
   $$(3 \times 9) \times (4 \times 6) = (3 \times __) \times (9 \times 6)$$

   d. Whenever zero is a factor, the product is _____.

2. Find the products.

   a. $3 \times 4 \times 6 =$

   b. $1 \times 8 \times 7 =$

   c. $2 \times 0 \times 3 =$

   d. $1 \times 2 \times 3 \times 4 =$

   e. $8 \times 2 \times 3 \times 5 =$

   f. $6 \times 9 \times 0 \times 3 =$

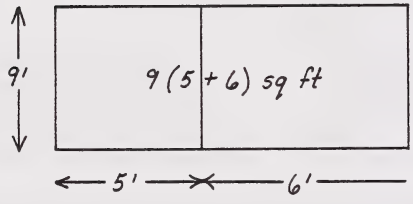
G. The Distributive Property of Multiplication Over Addition

We have studied the operations of addition and multiplication. These operations are related by the DISTRIBUTIVE PROPERTY. It tells us how we can distribute a multiplier over a series of addends.

EXAMPLE: Joe Foster was putting aluminum siding on the walls of his garage. The walls are nine feet high. Joe finished five feet of one wall when he was interrupted by a phone call. Returning, Joe covered another six feet before lunch. How much area did Joe cover before lunch?

Solution

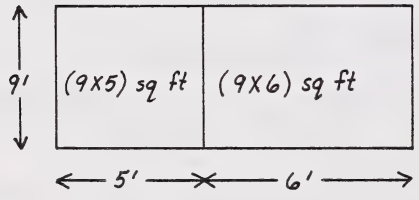
Method I



One way of finding the total area covered is to treat the portion completed as a whole and find the area of a rectangle 9 feet high and (5 + 6) feet long.

$$A = 9\ (5 + 6)$$
$$= 9\ (11)$$
$$= 99 \text{ sq. ft.}$$

Method II



The total area could also be found by calculating the area covered before the phone call and the area covered after the phone call and then finding the sum of the two areas. The total area is that of two rectangles 9' × 5' and 9' × 6'.

$$A = (9 \times 5) + (9 \times 6)$$
$$= 45 + 54$$
$$= 99 \text{ sq. ft.}$$

We have arrived at the same result by both methods; we can conclude that:

$$9\ (5 + 6) = (9 \times 5) + (9 \times 6)$$

This means that the product of the factor 9 and the factor (5 + 6) can be determined by distributing the factor 9 over the two addends 5 and 6.

In general, the distributive property says:

$a \times (b + c) = (a \times b) + (a \times c)$ A multiplier can be distributed over two addends.

We use the distributive property whenever we multiply factors that have more than one digit.

EXAMPLE: $6 \times 215 = \blacksquare$

Solution

Break 215 into units, tens, hundreds, and then apply the distributive property.

$6 \times 215 = 6\,(200 + 10 + 5)$

$= (6 \times 200) + (6 \times 10) + (6 \times 5)$

$= _____ + _____ + _____$

$= _____$

EXERCISE - USING THE DISTRIBUTIVE PROPERTY

1. Multiply the following factors horizontally. First break up the 2 or 3 digit factors into units, tens, etc. and then apply the distributive property.

   a. $7 \times 14 = 7 \times (10 + 4) = (7 \times ___) + (7 \times ___) = (____) + (____) = ____$

   b. $9 \times 281 = 9 \times (___ + ___ + ___) = (9 \times ___) + (9 \times ___) + (9 \times ___)$

   $= (____) + (____) + (____) = ____$

   c. $2 \times 193 = 2 \times (___ + ___ + ___) =$ _____

   $=$ _____ $=$ _____

   d. $5 \times 513 =$ _____

   _____

   e. $6 \times 918 =$ _____

   _____

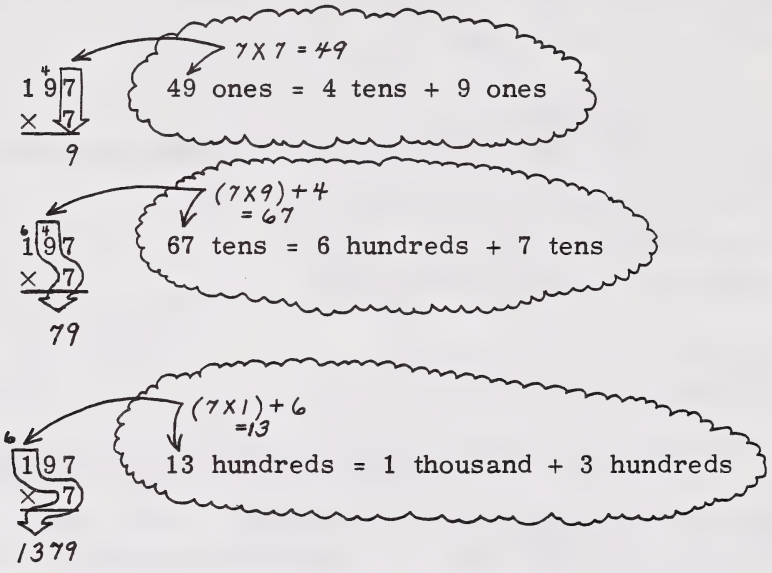
H. Multiplying Several Digits By One Digit

The place value of numbers that was discussed in Lesson One is important in multiplication.

As in addition, when we multiply we must "carry" a figure from one column to another.

EXAMPLE: Multiply 197 by 7.

Solution



Self-Correcting Exercise #2

Find the following products. Work as quickly as possible but aim for accuracy. The answers will be found on page 24 of this lesson. Check your own work. Redo any questions you had incorrect.

1. 224 × 4 = 896
2. 57 × 5
3. 64 × 3
4. 56 × 9
5. 156 × 4
6. 47 × 8
7. 255 × 6
8. 325 × 4

9. $\begin{array}{r} 217 \\ \times\ 5 \\ \hline \end{array}$

10. $\begin{array}{r} 446 \\ \times\ 3 \\ \hline \end{array}$

11. $\begin{array}{r} 2831 \\ \times\ 7 \\ \hline \end{array}$

12. $\begin{array}{r} 3971 \\ \times\ 6 \\ \hline \end{array}$

EXERCISE - MULTIPLYING SEVERAL DIGITS BY ONE DIGIT

Multiply.

1. $\begin{array}{r} 425 \\ \times\ 2 \\ \hline \end{array}$

2. $\begin{array}{r} 536 \\ \times\ 3 \\ \hline \end{array}$

3. $\begin{array}{r} 458 \\ \times\ 6 \\ \hline \end{array}$

4. $\begin{array}{r} 8543 \\ \times\ 7 \\ \hline \end{array}$

5. $\begin{array}{r} 4017 \\ \times\ 6 \\ \hline \end{array}$

6. $\begin{array}{r} 4582 \\ \times\ 5 \\ \hline \end{array}$

7. $\begin{array}{r} 319 \\ \times\ 3 \\ \hline \end{array}$

8. $\begin{array}{r} 672 \\ \times\ 4 \\ \hline \end{array}$

9. $\begin{array}{r} 841 \\ \times\ 8 \\ \hline \end{array}$

10. $\begin{array}{r} 7312 \\ \times\ 4 \\ \hline \end{array}$

11. $\begin{array}{r} 3239 \\ \times\ 6 \\ \hline \end{array}$

12. $\begin{array}{r} 5233 \\ \times\ 9 \\ \hline \end{array}$

13. $\begin{array}{r} 279 \\ \times\ 5 \\ \hline \end{array}$

14. $\begin{array}{r} 826 \\ \times\ 5 \\ \hline \end{array}$

15. $\begin{array}{r} 9135 \\ \times\ 6 \\ \hline \end{array}$

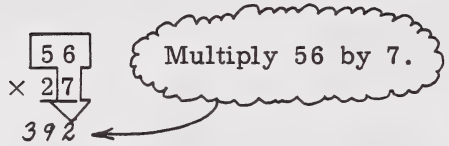
16. $\begin{array}{r} 58239 \\ \times\ 3 \\ \hline \end{array}$

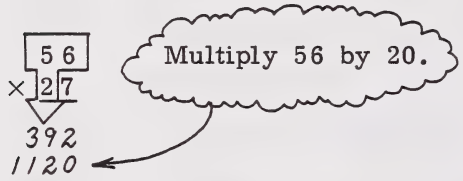
I. Multiplying By a Two-Digit Number

When you multiply by a factor with more than one digit it is usually necessary to list the partial products. When writing each partial product, keep in mind the place value of each digit.

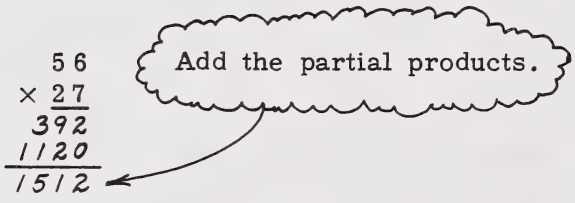
EXAMPLE: Multiply 56 by 27.

Solution





Care must be taken in this second step. Remember you are multiplying by 20. $56 \times 20 = 1120$. It is not necessary to write the zero if you are careful to begin the second partial product in the tens' column.



EXERCISE - MULTIPLYING BY A TWO-DIGIT NUMBER

Multiply. Show all calculations as in the solution to #1.

1. $\overset{4}{4}5 \times 19 = 405 + 45 = 855$

2. 86×37

3. 53×11

4. 38×21

5. $\begin{array}{r} 76 \\ \times\ 52 \\ \hline \end{array}$

6. $\begin{array}{r} 61 \\ \times\ 94 \\ \hline \end{array}$

7. $\begin{array}{r} 42 \\ \times\ 83 \\ \hline \end{array}$

8. $\begin{array}{r} 25 \\ \times\ 59 \\ \hline \end{array}$

9. $\begin{array}{r} 356 \\ \times\ 24 \\ \hline \end{array}$

10. $\begin{array}{r} 3175 \\ \times\ 84 \\ \hline \end{array}$

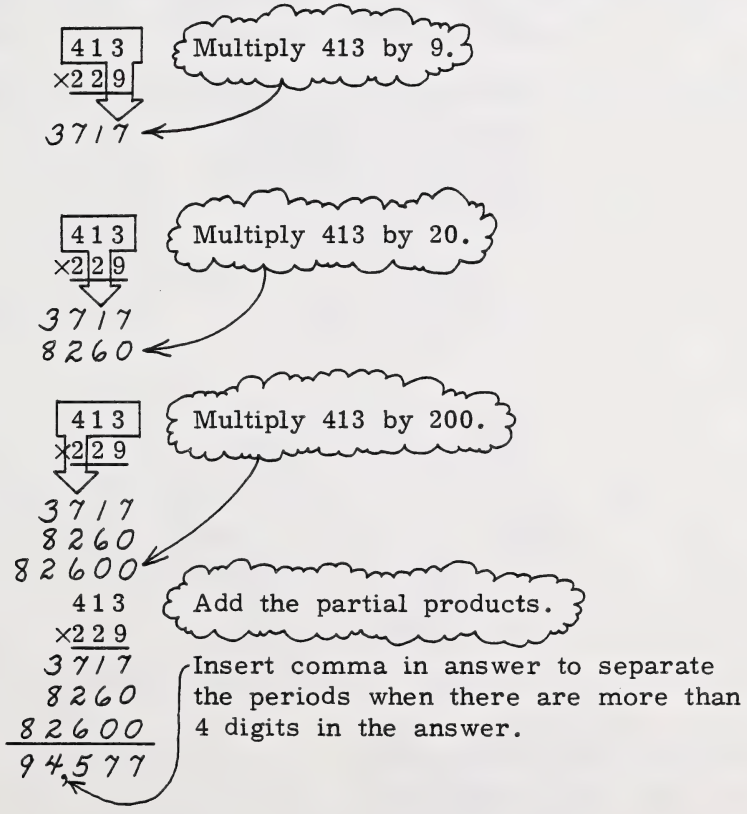
11. $\begin{array}{r} 5169 \\ \times\ 45 \\ \hline \end{array}$

12. $\begin{array}{r} 7032 \\ \times\ 93 \\ \hline \end{array}$

J. Multiplying By A Three-Digit Number

EXAMPLE: Multiply 413 by 229.

Solution



EXERCISE - MULTIPLYING BY A THREE-DIGIT NUMBER

Multiply. Show all calculations as in the solution to #1.

1. 523×312

1046
523
1569
$163,176$

2. 442×623

3. 753×173

4. 902×466

5. 5643×152

6. 4071×389

7. 928×576

8. 5833×979

9. 7652×219

10. 7238×526

11. 2192×317

12. 5301×272

K. The Zero Digit in Multiplication

Errors in multiplication are often made because of zeros in the multiplicand or multiplier. Study the following examples.

i. Zero as the Multiplicand or Multiplier

EXAMPLES:

$3 \times 0 = 0$

$$\begin{array}{r} 5280 \\ \times\ \ 0 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 0 \\ \times 1287 \\ \hline 0 \end{array}$$

$4 \times 2 \times 0 = 0$

$190 \times 0 \times 705 = 0$

$0 \times 54 = 0$

> Remember, whenever zero appears as a factor, the product is zero.

ii. Zero between the first and last digits of the multiplicand

EXAMPLE: Find the product of 9007 and 13.

Care must be taken to consider the place value of each partial product.

Solution

$$\begin{array}{r} 9\overset{2}{0}07 \\ \times\ \ 13 \\ \hline 27021 \end{array}$$

Multiply 9007 by 3.

$(3 \times 0) = 0$

$(3 \times 0) + 2$
$= 0 + 2$
$= 2$

$$\begin{array}{r} 9007 \\ \times\ \ 13 \\ \hline 27021 \\ 9007 \end{array}$$

Multiply 9007 by 10.

$1 \times 0 = 0$

$$\begin{array}{r} 9007 \\ \times\ \ 13 \\ \hline 27021 \\ 9007 \\ \hline 117,091 \end{array}$$

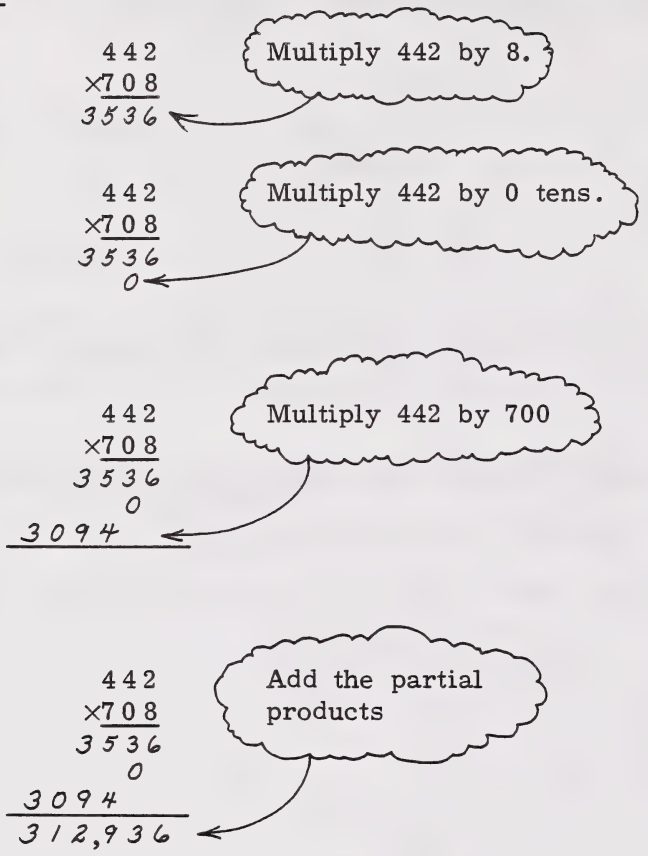
Add the partial products.

> Remember place value when you are carrying a digit from one column to the next higher column.

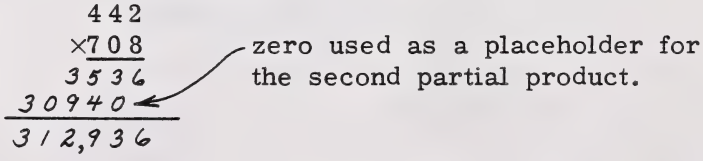
iii: Zero between the first and last digits of the multiplier.

EXAMPLE: Find the product of 442 and 708.

Solution



Notice that the second partial product consists of zeros and has no effect on the final product. In our work we can omit the second partial product, and use only one zero as a placeholder to insure that the remaining partial product has the correct place value.



Self-Correcting Exercise #3

Multiply. The answers will be found on page 24 of this lesson. Check your work. Redo any questions that you had incorrect.

1. 401×85

2. 37×207

3. 809×12

4. 207×36

5. 4003×72

6. 1702×504

7. 8012×15

8. 1823×607

iv. Zero at the end of the multiplicand or multiplier

When zeros appear at the end of either factor, multiply the factors together ignoring the zeros to the right of the last figure in both the multiplicand and multiplier. When the product of these two numbers is found, attach after the right-hand digit as many zeros as were ignored in both multiplicand and multiplier.

EXAMPLE: Find the product of 41500 and 530.

Solution

$$\begin{array}{rl} 415 & \text{Drop 2 zeros} \\ \times\ 53 & \text{Drop 1 zero} \\ 1245 & \\ 2075 & \\ \hline 21,995 & \text{Attach 3 zeros} \end{array}$$

Final product is 21,995,000

EXAMPLE: Find the product of 708 and 1000.

Solution
$$\begin{array}{r} 708 \\ \times\ 1 \\ \hline 708 \end{array}$$
Drop 3 zeros
Attach 3 zeros

Final product is 708,000

This example shows that the 3 zeros of the multiplier 1000 could have been attached to the right-hand side of the multiplicand to obtain the product. Use this method to do calculations mentally.

EXERCISE - THE ZERO DIGIT IN MULTIPLICATION

1. Fill in the blanks.

   a. When multiplying a number by 10, attach _____ to the multiplicand to find the final product.

   b. When multiplying a number by 100, attach _____ to the multiplicand to find the final product.

   c. When multiplying a number by 2000, multiply by 2 and attach _____ to find the final product.

   d. If multiplying a number by 800, multiply by _____ and attach _____ to find the final product.

2. Multiply. Use "short-cuts" whenever possible.

   a. $\begin{array}{r} 1250 \\ \times\ 300 \\ \hline 375,000 \end{array}$
   $125 \times 3 = 375$ Attach 3 zeros

   b. $\begin{array}{r} 1500 \\ \times\ 60 \end{array}$

   c. $\begin{array}{r} 1130 \\ \times\ 20 \end{array}$

d. 1000
× 100

e. 17
×200

f. 80
×100

g. 680
×100

h. 521
×10000

i. 3004
× 51

j. 702
×301

k. 1903
× 516

l. 1802
×7001

L. Checking Multiplication

An easy way to check multiplication is by interchanging the multiplicand and the multiplier. We can do this because the commutative law of multiplication tells us that the order in which we multiply two numbers does not affect the product.

EXAMPLE: Find the product of 423 and 201. Check your answer.

Solution

423
×201
423
8460
85,023

Check:

201
× 423
603
402
804
85,023

Which question is easier to do? _____
(left or right?)

If there is a zero digit in either of the factors, it is easier and shorter to use the number containing the zero digit as the multiplier.

EXERCISE - CHECKING MULTIPLICATION

1. Find the products. Check your work.

a. $\begin{array}{r} 38 \\ \times 86 \\ \hline \end{array}$ Check: $\begin{array}{r} 86 \\ \times 38 \\ \hline \end{array}$

b. $\begin{array}{r} 67 \\ \times 35 \\ \hline \end{array}$ Check:

c. $\begin{array}{r} 268 \\ \times\ 36 \\ \hline \end{array}$ Check:

d. $\begin{array}{r} 468 \\ \times 502 \\ \hline \end{array}$ Check:

2. Aim for accuracy. Find the products.

a. $\begin{array}{r} 7032 \\ \times\ 93 \\ \hline \end{array}$

b. $\begin{array}{r} 5169 \\ \times\ 45 \\ \hline \end{array}$

c. $\begin{array}{r} 9274 \\ \times\ 26 \\ \hline \end{array}$

d. $\begin{array}{r} 5480 \\ \times\ 71 \\ \hline \end{array}$

e. $\begin{array}{r} 356 \\ \times\ 24 \\ \hline \end{array}$

f. $\begin{array}{r} 2090 \\ \times\ 56 \\ \hline \end{array}$

g. 4286
× 93

h. 430
× 27

i. 9134
× 56

j. 493
× 29

k. 5678
× 87

l. 9876
× 91

m. 516
× 54

n. 18702
× 108

o. 29080
× 100

Key to Self-Correcting Exercises in Lesson 3

Exercise #1, page 3

1. 56	2. 54	3. 0	4. 1	5. 18
6. 56	7. 15	8. 36	9. 63	10. 64
11. 0	12. 48	13. 42	14. 21	15. 40

Exercise #2, page 12

1. 896	2. 285	3. 192	4. 504
5. 624	6. 376	7. 1530	8. 1300
9. 1085	10. 1338	11. 19,817	12. 23,826

Exercise #3, page 19

1. 401	2. 37	3. 809	4. 207
× 85	×207	× 12	× 36
2005	259	1618	1242
3208	740	809	621
34,085	7659	9708	7452
5. 4003	6. 1702	7. 8012	8. 1823
× 72	× 504	× 15	× 607
8006	6808	40060	12761
28021	85100	8012	109380
288,216	857,808	120,180	1,106,561

End of Lesson 3

Practical Mathematics
Module 1
Lesson 4

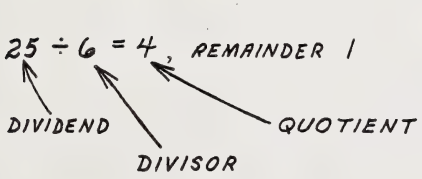
DIVISION OF WHOLE NUMBERS

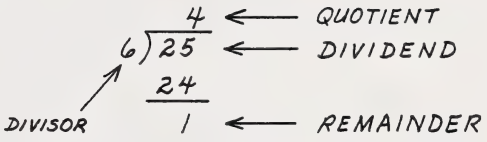
Division is the operation we use to separate one large group into a number of smaller groups. It can be thought of as repeated subtraction. For example, in the division question $8 \div 2 = \blacksquare$, we wish to divide a group of 8 objects into smaller groups containing 2 objects each. To do this we could subtract groups of 2.

$$\begin{array}{r} 8 \\ -2 \\ \hline 6 \\ -2 \\ \hline 4 \\ -2 \\ \hline 2 \\ -2 \\ \hline 0 \end{array}$$

A group of 8 objects contains 4 smaller groups of 2 objects with no objects remaining.

In division, the number being divided is called the DIVIDEND. The number that you divide by is called the DIVISOR. The result that is obtained is called the QUOTIENT, and the number that is left if the division is not exact is called the REMAINDER. The symbol ÷ is used to indicate the operation of division.





Division is the INVERSE OPERATION to multiplication.

We know that $2 \times 4 = 8$.

$2 = 8 \div 4$

and $4 = 8 \div 2$

Chart of Division Facts

	0	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

A. Basic Division Combinations

The chart on page 2 shows the basic combinations of division facts. To use the chart, find the dividend in the body of the table, and follow across the row to the left margin. The number at the left is the divisor, and the quotient will be the number at the top of the column in which the product is found.

Review the basic combinations by using the division table.

Self-Correcting Exercise #1

The answers are found on page 20 of this lesson. Correct your own work. If you have any errors, be sure to study the chart of basic facts and redo the questions you had incorrect.

1. $7\overline{)42}$ 2. $7\overline{)49}$ 3. $5\overline{)20}$ 4. $9\overline{)54}$

5. $3\overline{)27}$ 6. $8\overline{)0}$ 7. $7\overline{)21}$ 8. $6\overline{)42}$

9. $2\overline{)18}$ 10. $4\overline{)24}$ 11. $4\overline{)28}$

12. $64 \div 8 =$ _____ 13. $81 \div 9 =$ _____ 14. $36 \div 6 =$ _____

15. $48 \div 6 =$ _____ 16. $18 \div 6 =$ _____ 17. $48 \div 8 =$ _____

18. $45 \div 5 =$ _____ 19. $27 \div 9 =$ _____ 20. $28 \div 7 =$ _____

21. $56 \div 7 =$ _____ 22. $21 \div 3 =$ _____ 23. $36 \div 4 =$ _____

EXERCISE — BASIC DIVISION FACTS

1. Write the missing number or word in each sentence.

   a. In the number sentence 48 ÷ 8 = 6, the number _____ is the dividend; _____ is the divisor; and _____ is the quotient.

   b. In the number sentence 72 ÷ 9 = 8, the number 8 is the _____; 9 is the _____ and 72 is the _____.

   c. Write the division number sentence that uses 12 as a divisor, 3 as a quotient, and 36 as the dividend. _____

   Is there a remainder? _____

2. Tell whether the arrow points to the divisor, the dividend, the quotient, or the remainder. The first one is done for you.

   a. $3\overline{)12}$ (4) — quotient

   b. 19 ÷ 6 = 3 r 1 — _____

   c. $8\overline{)32}$ (4) — _____

   d. 49 ÷ 7 = 7 — _____

   e. 81 ÷ 9 = 9 — _____

   f. 56 ÷ 7 = 8 — _____

B. Two Methods of Division

EXAMPLE: 288 ÷ 4 = ■

$4\overline{)288}$

(70 × 4) → 280

8

(2 × 4) → 8

0

70 + 2 = 72 — quotient

$4\overline{)288}$ (72) — quotient

(70 × 4) → 280

8

(2 × 4) → 8

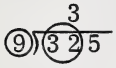
0

Both methods of division may be used, but when the divisor is small, we try to do as little writing as possible as in the second method. So in this course use the second method.

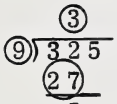
C. Dividing By a One-Digit Number

EXAMPLE: $325 \div 9 = \blacksquare$

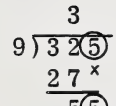
Solution



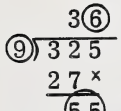
9 will not divide into 3, so we begin by dividing 9 into 32. 9 divides into 32 about 3 times. Place this partial quotient above the last digit in 32.



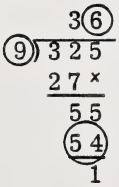
Multiply 9 by 3 and place the product 27 below 32. Subtract 27 from 32 and obtain a remainder of 5. Check to be sure that the remainder is less than the divisor.



Bring down the next digit in the dividend and place it to the right of the remainder, 5. (The small $\times$ under the 5 indicates that it has been brought down.)



Divide 9 into 55. It goes about 6 times. Place this partial quotient, 6, beside the first partial quotient, 3. (i.e. The quotient 36 appears above the 25 in the dividend.)



Multiply 9 by 6 and place the product 54 below 55. Subtract 54 from 55 and obtain a remainder of 1. (Again, check to make sure that your remainder is less than the divisor.) The division is complete since there are no more digits to bring down. (If there were, you would bring down these digits one at a time, and continue in the same manner.)

$325 \div 9 = 36$, remainder 1.

EXERCISE - DIVIDING BY A ONE-DIGIT NUMBER

1. $5\overline{)475}$ = 95

2. $4\overline{)724}$

3. $3\overline{)921}$

4. $2\overline{)829}$

5. $7\overline{)287}$

6. $9\overline{)819}$

7. $8\overline{)328}$

8. $3\overline{)936}$

9. $5\overline{)955}$

10. $7\overline{)608}$

11. $6\overline{)186}$

12. $7\overline{)4977}$

D. Division With Large Divisors

Study the following division example. The method is the same as explained in Section C: Dividing By a One-Digit Number (page 5.) Can you follow each step shown below?

EXAMPLE: 83387 ÷ 143 = ■

Solution

```
                583
          143) 83387
(500 × 143)-->715**
             1188
(80 × 143)-->1144
              447
(3 × 143)-->  429
               18
```

83387 ÷ 143 = 583, r 18

Self-Correcting Exercise #2

Work through each division problem and fill in the boxes as you go along. The answers will be found on page 20 of this lesson. Correct your work carefully.

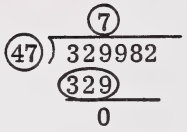
1.
```
      8□4
   47)38762
     3□6
     ---
     11□
      □4
     ---
     2□□
     18□
     ---
      □4
```

2.
```
      □5□4
   69)31284 6
     □76
     ---
     3□□
     □45
     ---
      □□4
      207
     ---
       2□6
       2□□
      ----
         □
```

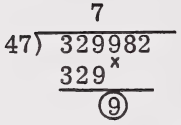
E. Using Zero As A Placeholder

EXAMPLE: Divide 329,982 by 47.

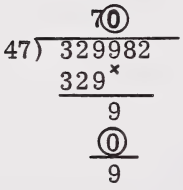
Solution



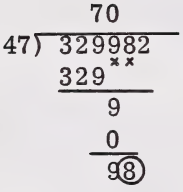
There is no remainder from the partial product 47×7.



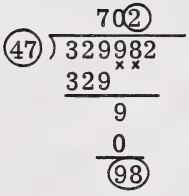
Bring down the next digit, 9, in the dividend.



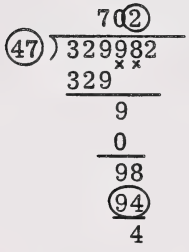
47 will not divide into 9 so we write a 0 above the 9 in the dividend. Then multiply $0 \times 47 = 0$ and write the partial product 0 below the 9. Subtract 9 - 0 = 9.



Bring down the next digit, 8 in the dividend. Place it alongside the 9 remainder.



Divide 47 into 98. It goes about twice. Place this partial quotient, 2, beside the placeholder 0. (The 2 should be directly above the 8 in the dividend.)



Multiply 47 by 2 and place the product 94 below 98. Subtract 94 from 98 and obtain the remainder 4.

```
      7020
47) 329982
    329
    ---
      9
      0
    ---
     98
     94
    ---
      42
       0
    ---
      42
```

47 will not divide into 42. To show this, use the placeholder 0 directly above the 2 in the dividend. You can continue and multiply $0 \times 47 = 0$, and place this partial product below the 42. If you subtract 42 - 0, the remainder is still 42.

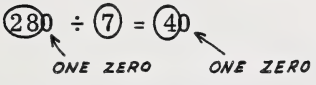
$329982 \div 47 = 7020$, remainder 42.

In division, zeros sometimes occur at the end of a dividend. We can often work out questions of this type quickly if we divide the non-zero digits and then add zeros to this quotient according to the number of zeros in the dividend.

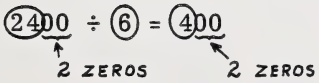
EXAMPLE: $640 \div 8 = \blacksquare$

Solution Our basic division facts tell us that $64 \div 8 = 8$. We can use this basic fact to help us solve the above problem. Since 640 is 10 times larger than 64, the quotient must be 10 times larger than the answer to $64 \div 8 = 8$.

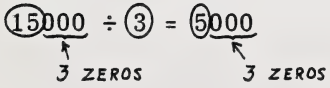
Now study the following examples:



(Think of the basic fact $28 \div 7 = 4$)



(Think of the basic fact $24 \div 6 = 4$)



(Think of the basic fact $15 \div 3 = 5$)

Zeros may also occur at the ends of both the divisor and dividend. In such cases, we can often divide the non-zero digits and then add zeros to this quotient according to the number of extra zeros in the dividend.

EXAMPLES:

180 ÷ 90 = 2	Think of the basic fact 18 ÷ 9 = 2. Since both 180 and 90 have one zero, the quotient is also 2.
3600 ÷ 60 = 60 (ONE EXTRA ZERO; ONE ZERO)	Think of the basic fact 36 ÷ 6 = 6. Since the dividend has one extra zero, you must add a zero to the quotient.
72,000 ÷ 80 = 900 (2 EXTRA ZEROS; 2 ZEROS)	Think of the basic fact 72 ÷ 8 = 9. Since the dividend has two extra zeros, you must add two zeros to the quotient.

EXERCISE - DIVISION

Divide. Work carefully. The first one is done for you.

1. $59\overline{)10830}$

$$\begin{array}{r} 183 \\ 59\overline{)10830} \\ 59\times\times \\ \hline 493 \\ 472 \\ \hline 210 \\ 177 \\ \hline 33 \end{array}$$

2. $24\overline{)1776}$

3. $55\overline{)3276}$

4. $41\overline{)2634}$

5. $37\overline{)2963}$

6. $98\overline{)8541}$

7. $18\overline{)4507}$

8. $19\overline{)1672}$

9. $94\overline{)9212}$

10. $91\overline{)873621}$

11. $341\overline{)8476321}$

12. $17\overline{)38692130}$

Divide. (Review examples at the bottom of page 9 and the top of page 10.)

1. $450 \div 9 =$ _____

2. $640 \div 8 =$ _____

3. $560 \div 8 =$ _____

4. $300 \div 5 =$ _____

5. $25,000 \div 500 =$ _____

6. $360 \div 90 =$ _____

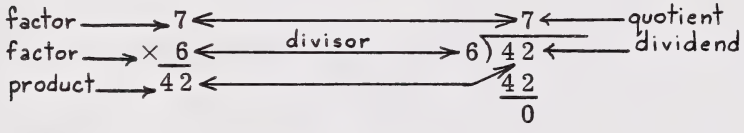
7. $3600 \div 60 =$ _____

8. $18000 \div 30 =$ _____

F. Checking Division

You will recall that on page 1 we said that division was the inverse operation to multiplication. Division can be used to find one factor when you know the product and another factor.

EXAMPLE: $7 \times 6 = 42$ $42 \div 6 = 7$



EXAMPLE: $8 \times \blacksquare = 56$ Find what number can be multiplied by 8 to get the product 56.

Solution

$$8\overline{)56}\quad 7$$

$$\begin{array}{r} 56 \\ \hline 0 \end{array}$$

Since $56 \div 8 = 7$ we know that $8 \times 7 = 56$.

EXERCISE - FINDING THE MISSING FACTOR

Use division to find the missing factors. The first one is done for you. Show your work.

		Calculations	Missing Factor
1.	$12 \times \square = 36$ (missing factor)	$12 \overline{)36}$ 3, 36, 0	3
2.	$\square \times 5 = 55$		
3.	$19 \times \square = 133$		
4.	$\square \times 17 = 357$		
5.	$\square \times 9 = 423$		

We can use this relationship between multiplication and division to check division questions. When the divisor goes evenly into the dividend, we can check the quotient obtained by multiplying the divisor and the quotient. The result should equal the dividend.

EXAMPLE: Divide 968 by 8 and check the answer.

Solution

```
          121  <-Quotient
Divisor --> 8) 968  <-Dividend
               8
               16
               16
                8
                8
                0  <-Remainder
```

Check:

```
  121  <-Quotient
x   8  <-Divisor
  968  <-Dividend
```

The product of the quotient and divisor does equal the dividend, so the division is correct.

When there is no remainder, DIVISOR × QUOTIENT = DIVIDEND

When there is a remainder involved, we can check the division by multiplying the divisor and quotient and adding to this product the remainder. The result should equal the dividend.

EXAMPLE: Divide 7278 by 46 and check the answer.

Solution

```
                158 <-- Quotient
Divisor --> 46) 7278 <-- Dividend
               46
               267
               230
                378
                368
                 10 <-- Remainder
```

Check:

```
 158 <-- Quotient
× 46 <-- Divisor
 948
632
7268
+ 10 <-- Add remainder
7278 <-- Dividend
```

The product of the quotient and divisor added to the remainder does equal the dividend, so the division is correct.

When there is a remainder, (DIVISOR × QUOTIENT) + REMAINDER = DIVIDEND.

EXERCISE - CHECKING DIVISION

1. Divide. Check your answer. The first one is done for you.

a.
```
      314
75) 23550
    225**
     105
      75
     300
     300
       0
```

Check:
```
  314
 X 75
1570
2198
23550
```

b. 24) 9648 Check:

c. $96\overline{)\,20160}$ Check:

d. $20\overline{)\,4060}$ Check:

2. Each of the following division questions contains an error. Circle the error, and then rework each question correctly.

a.
$$\begin{array}{r} 28 \\ 7\overline{)\,206} \\ 14 \\ \hline (5)6 \\ 56 \\ \hline \end{array}$$

$$\begin{array}{r} 29 \\ 7\overline{)\,206} \\ 14\,{}^{\times} \\ \hline 66 \\ 63 \\ \hline 3 \end{array}$$

b.
$$\begin{array}{r} 42 \\ 78\overline{)\,3176} \\ 302 \\ \hline 156 \\ 156 \\ \hline 0 \end{array}$$

c.
$$\begin{array}{r} 51 \\ 34\overline{)\,1635} \\ 170 \\ \hline 35 \\ 34 \\ \hline 1 \end{array}$$

d.
$$\begin{array}{r} 57\ r3 \\ 16\overline{)\,9123} \\ 80 \\ \hline 112 \\ 112 \\ \hline 3 \end{array}$$

3. (a) Divide 79635 by 319.

(b) Multiply the divisor by your quotient and to this product add the remainder.

G. Division With Zero

i. Zero as the Dividend (Number you are dividing into)

EXAMPLE: A band of 6 bank robbers agreed to share their "winnings" equally. But on a hold-up of the East-View Bank they were arrested before making a get-away. What was each robber's share of the money.

Solution Since the robbers were arrested, we assumed that they earned no money. If a total of 0 dollars is divided among 6 men, each man would receive 0 dollars.

$$0 \div 6 = 0$$

see part (ii) below

> When we divide zero by any number (except zero), we get a quotient of zero.
>
> $$0 \div \text{any number (except zero)} = 0$$

Use this rule to find the quotients below.

$0 \div 5 =$ 0 $\qquad 0 \div 176 =$ _____

$0 \div 8 =$ _____ $\qquad 0 \div 87 =$ _____

In every question above, the quotient is zero since the number you are dividing into (i.e. the dividend) is zero.

ii. Zero as the Divisor (Number you are dividing by)

EXAMPLE: A sum of \$6 is to be divided amongst 0 people. How much money does each person receive?

Solution Since the people who receive the money, do not exist, it is impossible to determine how much each of them receives.

$$6 \div 0 = \text{impossible}$$

> Division by zero is impossible.
>
> $$\text{any number} \div 0 = \text{impossible}$$

Use this rule to find the quotients below.

$5 \div 0 =$ impossible $\qquad 176 \div 0 =$ _____

$8 \div 0 =$ _____ $\qquad 87 \div 0 =$ _____

In every question above, the quotient is "impossible" since the number you are dividing by (i.e. the divisor) is zero.

EXERCISE - DIVISIONS WITH ZERO

1. Fill in the blanks.

   (a) If the number you are dividing <u>into</u> is zero, the quotient is _____.

   (b) If the number you are dividing <u>by</u> is zero, the quotient is _____.

   (c) A quotient equals zero if the _____ is zero.

   (d) A quotient is impossible to find when the _____ is zero.

   (e) Division by _____ is impossible.

2. Divide. (The quotient is <u>zero</u> if the number to the left of the division symbol is zero. The quotient is <u>impossible</u> if the number to the right of the division symbol is zero.)

   (a) $0 \div 10 =$ _____ (b) $0 \div 27 =$ _____

   (c) $10 \div 0 =$ _____ (d) $18 \div 0 =$ _____

   (e) $7 \div 0 =$ _____ (f) $0 \div 2 =$ _____

   (g) $0 \div 85 =$ _____ (h) $0 \div 79 =$ _____

   (i) $27 \div 0 =$ _____ (j) $100 \div 0 =$ _____

3. Fill in the blanks to make each statement true.

   (a) _____ $\div 14 = 0$ (b) $82 \div$ _____ = impossible

   (c) _____ $\div 0$ = impossible (d) $0 \div$ _____ $= 0$

   (e) 0 divided by 3 = _____ (f) 0 divided by 50 = _____

H. Word Problems

The problems in this section may involve addition, subtraction, multiplication or division. Be sure to write a number sentence, show your calculations and write your answer in a final statement. The first one is done for you.

1. A farmer took a truckload of 15 steers to market. The total weight of the steers was 6810 kg (kilograms). Assume all the steers were of equal weight. How much would one steer weigh?

$6810 \div 15 =$ ▨

$$\begin{array}{r} 454 \\ 15\overline{)6810} \\ \underline{60\times\times} \\ 81 \\ \underline{75} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

Each steer would weigh 454 kg.

2. Some castings for a lathe were made in a shop. The base weighed 190 kg; the pulleys, 7 kg; the tailstock, 12 kg; the headstock, 26 kg; the face plate, 2 kg; the bed, 100 kg. What was the total weight?

3. It is estimated that it requires 36 minutes to set up the work and cut the teeth on a gear blank. How many minutes should a foreman estimate will be required for cutting 12 gears?

4. A moving van is transporting furniture to a city 3945 km (kilometres) away. The driver expects to make the trip in 5 days. How many kilometres per day must he average?

5. An electrician cuts off the following lengths of wire: 128 feet, 46 feet, 51 feet, 36 feet, 40 feet, and 13 feet.

   (a) How much wire was used?

   (b) How much is left if the roll originally contained 600 feet?

6. An apprentice is asked to make 120 door stoppers. The actual time taken to make one stopper is 20 minutes. An additional minute is required to put the job in and take it out of the machine. What will be the labour cost if the apprentice receives $3 per hour?

Key to Self-Correcting Exercises in Lesson 4

Exercise #1, page 3

1. 6	2. 7	3. 4	4. 6
5. 9	6. 0	7. 3	8. 7
9. 9	10. 6	11. 7	12. 8
13. 9	14. 6	15. 8	16. 3
17. 6	18. 9	19. 3	20. 4
21. 8	22. 7	23. 9	

Exercise #2, page 7

1.
```
      824
47) 38762
    376xx
    ----
     116
      94
     ---
      222
      188
     ---
       34
```

2.
```
      4534
69) 312846
    276xxx
    ------
      368
      345
     ---
       234
       207
      ---
        276
        276
       ---
          0
```

End of Lesson 4

Practical Mathematics
Module 1
Lesson 5

WORKING WITH WHOLE NUMBERS

Topic One: Rounding Whole Numbers

How precise are you in your speech? Do you say it took you 86 minutes and 47 seconds to do Lesson 3, or do you say it took you about an hour and a half. (90 minutes)? Would you tell a friend that the distance between Calgary and Edmonton was 298 km or about 300 km? The ROUNDING OF NUMBERS is frequently used when precision is not required.

There are some simple rules that help to make the rounding of numbers systematic.

A. Rounding to the nearer ten

In rounding to the nearer tens digit, look at the digit in the ones place of the numeral. If it is 5, 6, 7, 8, or 9 then

(1) replace the tens digit with the next greater digit and
(2) replace the ones digit with a zero.

However, if the digit in the ones place is 4, 3, 2, or 1 then

(1) leave the tens digit unchanged and
(2) replace the ones digit with zero.

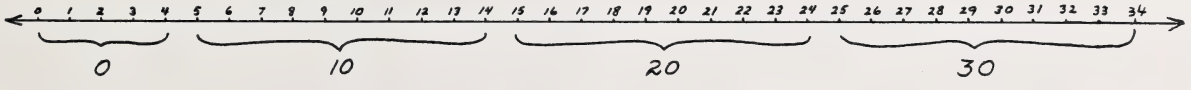
EXAMPLES: Round to the nearer tens digit.

(1) 3(2)
↖Look at the ones digit. Since it is 2 we leave the tens digit unchanged and replace the ones digit with zero.
So 32 rounded to the nearer tens place is 30.

(2) 1 5(7)
↖Look at the ones digit. Since it is 7, we replace the tens digit (5) with the next greater digit (6), and replace the ones digit with zero.

So 157 rounded to the nearer ten is 160.

The diagram below illustrates rounding to the nearer 10.



All numbers rounded to the nearer ten have a zero in the ones place.

Self-Correcting Exercise #1

Look carefully at the numbers below. Round each to the nearer ten. The answers will be found on page 24 of this lesson. Check your own work and correct any errors that you may have.

1.	103	_____	7.	1056	_____
2.	4	_____	8.	989	_____
3.	42	_____	9.	499	_____
4.	76	_____	10.	601	_____
5.	15	_____	11.	791	_____
6.	23	_____	12.	1807	_____

B. Rounding to the nearer hundred

If you are to round a number to the nearer hundred, the digit to look at is in the tens place in the numeral. If the value of the digit is 5 or greater than 5, then replace the hundreds digit with the next greater digit, placing zeros in both the tens place and the ones place. If the value of the tens digit is less than 5, leave the hundreds digit as it is and place zeros in both the tens place and the ones place.

EXAMPLES: Round to the nearer hundreds digit.

(1) 3057 — Look at the tens digit. Since it is 5, we replace the hundreds digit (0) with the next greater digit (1) and replace the tens and the ones digits with zeros.

3057 rounded to the nearer hundred is 3100.

(2) 949 — Look at the tens digit. Since it is 4, we leave the hundreds digit (9) unchanged and replace the tens and the ones digit with zeros.

949 rounded to the nearer hundred is 900.

All numbers rounded to the nearer hundred have zeros in both the ones place and the tens place.

Self-Correcting Exercise #2

Look carefully at the numbers below. Round each to the nearer hundred. The answers will be found on page 24 of this lesson. Check your own work and correct any errors that you may have.

1.	859	_____	7.	5495	_____
2.	409	_____	8.	777	_____
3.	1236	_____	9.	3052	_____
4.	1342	_____	10.	155	_____
5.	529	_____	11.	280	_____
6.	1852	_____	12.	1573	_____

C. Rounding to the nearer thousand

You should now be able to write the method of rounding a whole number to the nearer thousand. Fill in the blanks.

If you are to round a number to the nearer thousand, the digit to look at is in the _____ place in the numeral. If the value of the digit is five or _____, then replace the _____ digit with the next greater digit, placing zeros in the _____, _____ and _____ places. If the value of the hundreds digit is less than _____, leave the _____ digit unchanged and place _____ in the hundreds, tens and ones places.

All numbers rounded to the nearer thousand have zeros in the ones place, tens place, and hundreds place.

Self-Correcting Exercise #3

Look carefully at the numbers below. Round each to the nearer thousand. The answers will be found on page 24 of this lesson. Check your own work and correct any errors that you may have.

1.	10,763	_____	7.	9052	_____
2.	8923	_____	8.	5663	_____
3.	9532	_____	9.	4572	_____
4.	17,533	_____	10.	40,321	_____
5.	44,293	_____	11.	15,129	_____
6.	82,601	_____	12.	7653	_____

D. Rounding Quotients in Division

While some division questions work out evenly, others have remainders. In cases in which remainders occur, we sometimes wish to express the quotient to the nearer whole number. To do this we must decide if the remainder is less than half the divisor, exactly half the divisor, or more than half the divisor and adjust the quotient accordingly.

(1) If the remainder is less than half the divisor, we leave the quotient unchanged.

(2) If the remainder is half the divisor or more than half the divisor, we increase the quotient by 1.

EXAMPLES: Divide. Express answers to nearer whole number.

(1)
$$\begin{array}{r} 17 \\ 7\overline{)123} \\ \underline{7\times} \\ 53 \\ \underline{49} \\ (4) \end{array}$$

Since $2 \times 4 = 8$, which is greater than the divisor 7, the remainder is more than half the divisor. Thus, the quotient should be increased by 1.

$123 \div 7 =$ 18, to the nearer whole number.

(2)
$$\begin{array}{r} 6 \\ 46\overline{)298} \\ \underline{276} \\ (22) \end{array}$$

Since $2 \times 22 = 44$, which is less than the divisor 46, the remainder is less than half the divisor. Thus, the quotient should be left unchanged.

$298 \div 46 =$ 6, to the nearer whole number.

Self-Correcting Exercise #4

Divide. Express answers to nearer whole number. Answers may be found on page 25 of this lesson.

1. $12\overline{)\ 186}$

$186 \div 12 =$ _____

(to nearer whole number)

2. $8\overline{)\ 4802}$

$4802 \div 8 =$ _____

(to nearer whole number)

3. $26\overline{)\ 2692}$

$2692 \div 26 =$ _____

(to nearer whole number)

4. $509\overline{)\ 57,060}$

$57,060 \div 509 =$ _____

(to nearer whole number)

EXERCISE - ROUNDING WHOLE NUMBERS

1. Fill in the chart. The first one is done for you.

		ROUNDED TO NEARER TEN	ROUNDED TO NEARER HUNDRED	ROUNDED TO NEARER THOUSAND
1.	1739	1740	1700	2000
2.	20,735			
3.	91,244			
4.	5760			
5.	14,526			
6.	9991			
7.	8723			
8.	45,628			
9.	723,401			
10.	197,259			
11.	1,804,523			
12.	9,478,235			

2. At a recent moto-cross race held at the newly opened Trail Hill Park, the announcer reported that 4500 people were in attendance. The paid attendance was 4486. To what place had the announcer rounded the attendance?

_____

3. When planning a trip from Whitecourt to Waterton Park, Ian found that the trip was about 736 km. In calculating the approximate driving time, to what place should Ian round the distance?

_____

_____

4. Divide. Round each answer to nearer whole number.

(a) $18\overline{)\ 95,535}$ (b) $207\overline{)\ 62,409}$

Rounded quotient is _____. Rounded quotient is _____.

5. A man travels 2045 km at an average rate of 90 km/h. Find the number of hours it takes him to make the journey, to the nearer whole number. (Write a number sentence and show your calculations when solving word problems.)

6. Five children are to divide an inheritance of $825,432 equally amongst themselves. How much money should each child receive to the nearer dollar?

Topic Two: Averaging Whole Numbers

You are probably already familiar with the idea of averages. In school if you had several examinations or tests up to a certain point you would probably be interested in knowing how you stood. In sports, players are rated with averages which tell their proficiency at the game. Statistics published each year tell us the average annual income for Canadians. Averages are used when we wish to pick a single number to describe a collection of numbers.

To calculate an average, add up all the numbers you have in the particular collection, and then divide this total by the number of values you have in the collection.

EXAMPLE: Craig Jones obtained the following grades on his final examinations: Mathematics 69%, Literature 54%, Chemistry 62% Social Studies 60%, Biology 65%, and Physics 60%. What was his average grade in the 6 subjects, to the nearer whole percent.

Solution

Total marks for the examinations:

$$\begin{array}{r} 69 \\ 54 \\ 62 \\ 60 \\ 65 \\ 60 \\ \hline 370 \end{array}$$

$$\text{Average grade} = \frac{\text{Total marks}}{\text{Number of Examinations}} = \frac{370}{6} = 61, \text{ r } 4 = 62\% \text{ (to nearer whole percent)}$$

Craig's average grade was 62%.

EXAMPLE: Carmen Liddel purchased gasoline at four different locations last month. At the first station she paid 78¢ a gallon, at the second she paid 81¢ a gallon, at the third she paid 84¢ a gallon and at the fourth 80¢ a gallon. What was the average price per gallon of the gasoline she bought last month? (Give answer to nearer cent.)

Total Cost per gallon:

$$\begin{array}{r} 78 \\ 81 \\ 84 \\ 80 \\ \hline 323 \end{array}$$

$$\text{Average Cost per gallon} = \frac{323}{4} = 80, \text{ r } 3 = 81¢ \text{ (to nearer cent)}$$

The average cost per gallon was 81¢.

An AVERAGE is the quotient obtained when a series of numbers is added together and their sum divided by the number of items in the series.

EXERCISE - AVERAGING WHOLE NUMBERS

1. In an interest class teaching the fundamental techniques of the martial art of kung-fu there were 11 students enrolled. 1 student was 10 years old, 2 students were 19 years old, 1 student was 22 years old, 2 students were 23 years old, 1 student was 24 years old, 3 students were 27 years old and 1 student was 35 years old. What was the average age of the students in the class, to the nearer whole number?

   Total age in years $= 10 + (19 \times 2) + 22 + (23 \times 2) + 24 + (27 \times 3) + 35$

   $=$

   $=$

   Average age $= \dfrac{\text{Total age in years}}{\text{Number of students}} = \dfrac{\qquad}{\qquad} =$

   Concluding Statement: _____

2. Miss Jones told her students that anyone whose average is 80 or better in mathematics will not have to write the final exam in that subject. Steve's grades in mathematics throughout the year were 83, 76, 82, 71, 89, 77 and 82. Will Steve have to write the final exam?

   Concluding Statement: _____

3. The number of farms and the total area of farmland in each Canadian Province are given below. Fill in the last column giving the average acreage per farm, to the nearer whole number. Show your calculations in the space provided at the bottom of the page.

Province	Number of Farms	Area in Acres	Average Acres Per Farm
Newfoundland	3,387	71,814	21
P.E.I.	9,432	1,065,463	
Nova Scotia	21,075	2,775,642	
New Brunswick	22,116	2,981,449	
Quebec	122,617	15,910,128	
Ontario	140,602	19,879,646	
Manitoba	49,201	17,931,817	
Saskatchewan	103,391	62,793,979	
Alberta	79,424	45,970,395	
British Columbia	24,748	4,538,881	
Territories	22	4,477	

Show your calculations here.

4. The following table shows the electrical switching units completed in one week at an assembly plant:

6	men working together	completed	24 units
8	men working together	completed	31 units
13	men working together	completed	38 units
21	men working together	completed	45 units
23	men working together	completed	49 units
19	men working together	completed	56 units
14	men working together	completed	59 units
11	men working together	completed	63 units
2	men working together	completed	67 units

What is the average number of units finished per man per week? Give answer to nearer whole number.

Concluding Statement: _____

Topic Three: Factors and Multiples

A. Factors

In Lesson 3, "factors" were described as the numbers that were used or could be used to make the product. The multiplicand and the multiplier are two special "factors" of the product, but it is possible for the product to have other factors.

EXAMPLE: $3 \times 6 = 18$

The multiplicand (3) and the multiplier (6) are two factors of 18.

There are other ways we can multiply numbers together to get 18:
$2 \times 9 = 18$
$1 \times 18 = 18$

We can take 18 apart and find its various factors, or the numbers by which it can be divided exactly.

The number 18 has 6 factors: 1, 2, 3, 6, 9, 18.

EXAMPLE: What are the factors of 21?

Solution

We know that $1 \times 21 = 21$
$3 \times 7 = 21$

So the factors of 21 are 1, 3, 7 and 21 because these numbers will divide 21 exactly.

EXERCISE - FACTORS

1. List the factors of the following. The first one is done for you.

a. 24

$1 \times 24 = 24$
$2 \times 12 = 24$
$3 \times 8 = 24$
$4 \times 6 = 24$
Factors are 1, 2, 3, 4, 6, 8, 12, 24

b. 19

c. 38

d. 49

e. 75

f. 36

B. Prime and Composite Numbers

PRIME NUMBERS are numbers that are divisible by only themselves and 1. Memorize the following prime numbers that are less than 30.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

COMPOSITE NUMBERS are numbers that have divisors other than themselves and 1. The following numbers are all composite numbers that are less than 30. Learn to recognize them as being composite rather than prime.

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28

Any composite number can be written as the product of prime numbers. To do this, we divide it by one of its prime factors so it can be expressed as the product of two factors. If the other factor is not a prime number, we must divide it by one of its prime factors and express it as a product. We continue this process until all the factors involved are prime numbers.

EXAMPLE: Write 84 as a product of prime factors.

Solution

2 is a prime number and is a factor of 84. We can divide 84 by 2 in order to find the other factor.

i.e.
$$2\overline{)\,84}\quad 42$$
$$8^{\times}$$
$$04$$
$$4$$
$$0$$

$\therefore 84 = 2 \times 42$

Since 42 is not a prime number, we must divide it by one of its prime factors. We can divide by 2 since 2 is a prime number and is a factor of 42.

i.e.
$$2\overline{)\,42}\quad 21$$
$$4^{\times}$$
$$02$$
$$2$$
$$0$$

$\therefore 84 = 2 \times (2 \times 21)$ — These are factors of 42.

But, 21 is not a prime number. The smallest prime number that will divide into 21 is 3.

i.e.
$$3\overline{)\,21}\quad 7$$
$$21$$
$$0$$

$\therefore 84 = 2 \times 2 \times (3 \times 7)$ — These are factors of 21

84 has now been expressed as a product of primes since 2, 3, and 7 are all prime numbers. (See list of prime numbers at the top of this page.)

NOTE: You can check your work by multiplying the prime factors to make sure the product is 84.

i.e. $84 = 2 \times 2 \times 3 \times 7 = 4 \times 3 \times 7 = 12 \times 7 = (84)$ ← given number

EXERCISE - PRIME FACTORS

1. For each number, tell if it is a prime or composite. For each composite number, express it as the product of its two prime factors. The first two are done for you.

a.	35	composite	$35 = 5 \times 7$
b.	23	prime	———
c.	26		
d.	57		
e.	58		
f.	59		
g.	71		
h.	33		
i.	73		
j.	101		

2. Find the prime factors. The first one is done for you. (All the final factors you list must be divisible by only themselves and 1.)

a. 210
$= 2 \times 105$
$= 2 \times (3 \times 35)$
$= 2 \times 3 \times (5 \times 7)$

b. 22

c. 60

d. 36

e. 84

f. 92

g. 380

h. 712

i. 195

C. Highest Common Factor (H.C.F.)

The HIGHEST COMMON FACTOR of two or more numbers is the largest number that will divide into the given numbers. For example, the H.C.F. of 8 and 12 is 4 since 4 is the largest number that will divide into both 8 and 12. Sometimes the H.C.F. is hard to determine at a glance. We can then use the following procedure.

1. Express each number as the product of primes.
2. Pick out all those prime factors that are common to all the numbers.
3. Find the product of these common factors. This product is the H.C.F. of the numbers.

EXAMPLE 1: Find the H.C.F. of 66 and 165

Solution

We can use the method outlined on page 13 to express each number as the product of primes.

$$66 = 2 \times 33 = 2 \times (3 \times 11) \qquad 165 = 3 \times 55 = 3 \times (5 \times 11)$$

Note that the factors 3 and 11 appear in both factorizations.

$$66 = 2 \times \boxed{3} \times \boxed{11}$$
$$165 = \boxed{3} \times 5 \times \boxed{11}$$

common factors

The H.C.F. is the product of these common factors.

$$\text{i.e. H.C.F.} = 3 \times 11 = \underline{33}$$

EXAMPLE 2: Find the H.C.F. of 44, 60, and 72.

Solution

$$44 = \boxed{2} \times \boxed{2} \times 11$$
$$60 = \boxed{2} \times \boxed{2} \times 3 \times 5$$
$$72 = \boxed{2} \times \boxed{2} \times 2 \times 3 \times 3$$

Common factors

Work out these prime factorizations on another sheet of paper. (Use method given on page 13.)

The factor 2 appears twice in all three factorizations.

Hence,

$$\text{H.C.F.} = 2 \times 2 = \underline{4}$$

EXERCISE - HIGHEST COMMON FACTOR

Find the highest common factor in each exercise.

1. 18, 24

18 = ___ × ___ × ___

24 = ___ × ___ × ___ × ___

H.C.F. = ___ × ___ = ___

2. 60, 96

60 = ___ × ___ × ___ × ___

96 = ___ × ___ × ___ × ___ × ___ × ___

H.C.F. = ___ × ___ × ___ =

3. 72, 63

4. 36, 54

5. 28, 70

6. 18, 76, 95

7. 24, 36, 56

8. 64, 92, 128, 184

D. Lowest Common Multiple (L.C.M.)

i. What is a Multiple?

> A MULTIPLE of a given number is the product of that number and another non-zero whole number factor.

EXAMPLES:

12 is a multiple of 3 since $12 = 3 \times (4)$ ← non-zero whole number factors

65 is a multiple of 13 since $65 = 13 \times (5)$

We can check whether a given number "a" is a multiple of another number "b" by dividing "b" into "a".

1. If "b" divides evenly into "a", then "a" is a multiple of "b".
2. If "b" does not divide evenly into "a", then "a" is not a multiple of "b".

EXAMPLES:

1. 20 is a multiple of 4 since $20 \div 4 = 5$.
2. 20 is not a multiple of 3 since $20 \div 3 = 6$, remainder 2.
3. 6 is not a multiple of 12 since $6 \div 12$ = no whole number.

Is 12 a multiple of 4? _____ Is 18 a multiple of 4? _____

Is 5 a multiple of 10? _____ Is 96 a multiple of 8? _____

A given number has an infinite number of multiples. We can find these multiples by multiplying the number by the non-zero whole numbers 1, 2, 3, 4, 5, 6, 7, and so on.

EXAMPLE: We can find the multiples of 3 by multiplying 3 by the non-zero whole numbers 1, 2, 3, 4, 5, 6, 7, and so on.

3	3	3	3	3	3	3	
$\times 1$	$\times 2$	$\times 3$	$\times 4$	$\times 5$	$\times 6$	$\times 7$	etc.
3	6	9	12	15	18	21	

Therefore, the multiples of 3 are 3, 6, 9, 12, 15, 18, 21, and so on.

Find the first five multiples of 7. 7, ___, ___, ___, 35

Find the first four multiples of 11. ___, ___, ___, ___

Find a number that is a multiple of both 2 and 3. _____

In general, a product is said to be a multiple of any of its factors.

EXAMPLE: The factors of 12 are 1, 2, 3, 4, 6, and 12. Therefore, 12 is a multiple of 1, 2, 3, 4, 6, and 12.

The number 30 is a multiple of what numbers? _____

(ii) Lowest Common Multiple (L.C.M.)

The L.C.M. of two or more numbers is the smallest number that is a multiple of all the numbers.

Study the following examples to see how the L.C.M. of a set of numbers can be determined.

EXAMPLE I: Find the L.C.M. of 8, 12, and 4.

Solution 1 We could find the L.C.M. by listing multiples of each number and then finding the lowest multiple that is common.

Multiples of 8	Multiples of 12	Multiples of 4
$8 \times 1 = 8$	$12 \times 1 = 12$	$4 \times 1 = 4$
$8 \times 2 = 16$	$12 \times 2 = 24$	$4 \times 2 = 8$
$8 \times 3 = 24$	$12 \times 3 = 36$	$4 \times 3 = 12$
$8 \times 4 = 32$	$12 \times 4 = 48$	$4 \times 4 = 16$
etc.	etc.	$4 \times 5 = 20$
		$4 \times 6 = 24$
		etc.

We see that 24 is the smallest multiple that is common to all 3 numbers.

Use this method to find the L.C.M. of:

1. 9 and 15.

   Multiples of 9 are 9, ___, 27, ___, ___, 54, ...

   Multiples of 15 are 15, 30, ___, ___, ___, ...

   The L.C.M. is _____

2. 3, 6, and 8.

   Multiples of 3 are ___, ___, 9, ___, ___, 18, ___, ___, 27, ...

   Multiples of 6 are 6, ___, 18, ___, 30, ___, ...

   Multiples of 8 are ___, 16, ___, ___, ___, ...

   The L.C.M. is _____

3. 10 and 12.

   Multiples of 10 are _____

   Multiples of 12 are _____

   The L.C.M. is _____

OR

Solution 2

We could write the prime factors of each number and then list each prime factor the greatest number of times it occurs in any one number. The product of these factors we have listed is the L.C.M.

Prime Factors of each number:

$8 = 2 \times 2 \times 2$
$12 = 2 \times 2 \times 3$
$4 = 2 \times 2$

The prime factor 2 occurs 3 times in the number 8.
The prime factor 3 occurs once in the number 12.

Therefore, the L.C.M. must contain the factor 2 used 3 times and the factor 3 used once.

i.e. L.C.M. $= \underbrace{2 \times 2 \times 2}_{\text{3 factors}} \times 3 = 24$

Thus, the L.C.M. of 8, 12, and 4 is 24.

EXAMPLE II: Find the L.C.M. of 14 and 9.

Solution

$14 = 2 \times 7$
$9 = 3 \times 3$

L.C.M. $= 2 \times 7 \times \underbrace{3 \times 3}_{\text{2 factors}} = 126$

NOTE: Since 14 and 9 have no common prime factors, their L.C.M. is 14×9 or 126.

EXAMPLE III: Find the L.C.M. of 3, 50, and 125.

Solution

3 ←prime number
$50 = 2 \times 5 \times 5$
$125 = 5 \times 5 \times 5$

The prime factor 3 occurs once in the number 3.
The prime factor 2 occurs once in the number 50.
The prime factor 5 occurs 3 times in the number 125.

L.C.M. $= 3 \times 2 \times \underbrace{5 \times 5 \times 5}_{\text{3 factors}} = 750$

EXERCISE - LOWEST COMMON MULTIPLE

Find the L.C.M. of each of the following sets of numbers. The first one is done for you.

1. 3, 7, 10

3 ← prime number
7 ← prime number
10 = 2 x 5

L.C.M. = 3 x 7 x 2 x 5
= 210

2. 4, 6, 10

4 = _____ × _____

6 = _____ × _____

10 = _____ × _____

L.C.M. = (2 × 2) × _____ × _____

= _____

Since 2 appears twice in the factorization of 4, it must appear twice in the L.C.M.

3. 5, 6, 9

5 ← prime number

6 = _____ × _____

9 = _____ × _____

L.C.M. = ___ × ___ × ___ × ___

= _____

4. 4, 8, 16

4 = ___ × ___

8 = ___ × ___ × ___

16 = ___ × ___ × ___ × ___

L.C.M. = ___ × ___ × ___ × ___

= ___

5. 3, 15, 18

3 ← prime number

15 = _____ × _____

18 = _____ × _____ × _____

L.C.M. =

6. 8, 10, 14

8 =

10 =

14 =

L.C.M. =

7. 3, 7, 21

8. 60, 12, 3

9. 9, 5

10. 7, 49, 149

Topic Four: Estimating Answers

For many math questions it would be helpful if you had a general idea of what the answer would be to a problem before you actually worked it out. In this way, careless errors and unreasonable answers could often be avoided. Using rounded numbers is very convenient in estimating reasonable answers in calculations.

EXAMPLE: The trip from Coutts to Edson is 789 km. If Brian drives at a steady rate of 99 km/h, how much time will he spend driving?

Solution:

It is best to estimate your answer first. If the distance of 789 km is rounded to the nearer hundred, it becomes 800 km. If the rate of 99 km/h is rounded to the nearer 10, it becomes 100 km/h. Thus, a reasonable estimate of driving time would be:

$800 \div 100$ or approximately 8 hours.

REMEMBER THAT $800 \div 100$ IS EQUIVALENT TO $8 \div 1$ (SEE P. 10, LESSON 4)

EXAMPLE: A pressure switch on an air pump is set so that the motor begins to operate at 46 pounds and is shut off at 297 pounds. By how many pounds does the pressure in the tank vary? Give an estimate of your solution.

Solution Usually estimations can be done mentally. That is their advantage — you can calculate quickly and obtain an approximate answer. Mentally, we would round the numbers to the nearer ten. Our mental calculations would then be 300 - 50 = 250. The pressure in the tank varies by about 250 pounds.

EXERCISE — ESTIMATING ANSWERS

For each of the following problems, give an estimate of the final solution. Show your calculations with the numbers you have rounded. The first one is done for you.

1. The social committee for the local community league was selling tickets to their next party. Each person was to be responsible for selling 18 tickets. If there were 9 volunteers willing to sell tickets, how many tickets would be sold?

   For an estimation of the answer we should round 18 to 20, and 9 to 10.

   Then we would have:
   $$\begin{array}{r} 20 \\ \times\ 10 \\ \hline 200 \end{array}$$

   About 200 tickets should be sold.

2. Bob was transferred from Edmonton to Kamloops, B.C., a distance of 840 km. He decided to drive from Edmonton to Nelson to visit friends and then proceed to Kamloops. The distance from Edmonton to Nelson is 930 km, and the distance from Nelson to Kamloops is 460 km.

   a. How far did Bob drive altogether?

b. How much farther did he have to drive to visit his friends in Nelson than if he had travelled directly from Edmonton to Kamloops?

3. Mr. Jones paid $118.50 for a set of four tires. What was the price per tire?

4. Charlene, a nurse at a downtown clinic, earns a take-home pay of $723 per month. Her expenses per month include: rent - $162; utilities - $8; groceries - $46; gas $12; loan payment - $108; clothing - $17; entertainment - $36 and miscellaneous spending - $24.

a. What are her total expenses per month?

b. How much can she save each month towards her holidays in Europe?

5. Gordon and Mark were planning a party. If everyone came there would be a total of 23 people at the party. They planned to buy 12 cases of refreshments. If each case holds 12 bottles, how many bottles of refreshments would each person have? (Assume everyone receives the same amount.)

Answers to Self-Correcting Exercises in Lesson 5

Exercise #1, page 2

1. 100
2. 0
3. 40
4. 80
5. 20
6. 20
7. 1060
8. 990
9. 500
10. 600
11. 790
12. 1810

Note that in all these numbers, there is a 0 in the ones place.

Exercise #2, page 3

1. 900
2. 400
3. 1200
4. 1300
5. 500
6. 1900
7. 5500
8. 800
9. 3100
10. 200
11. 300
12. 1600

Note that in all these numbers the last 2 digits are zeros.

Exercise #3, page 4

1. 11,000
2. 9000
3. 10,000
4. 18,000
5. 44,000
6. 83,000
7. 9000
8. 6000
9. 5000
10. 40,000
11. 15,000
12. 8000

Note that in all these numbers the last 3 digits are zeros.

Exercise #4, page 5

1.
```
      15
  12)186
     12
     66
     60
      6 ← Half the divisor
```

186 ÷ 12 = 16 ← (Last digit was increased by 1.)

2.
```
     600
   8)4802
     48
     00
      0
      02
      0
      2 ← Less than half the divisor
```

4802 ÷ 8 = 600 ← (Last digit was left unchanged.)

3.
```
     103
  26)2692
     26
     09
      0
     92
     78
     14 ← More than half the divisor
```

2692 ÷ 26 = 104 ← (Last digit was increased by 1.)

4.
```
     112
 509)57060
     509
     616
     509
     1070
     1018
       52 ← Less than half the divisor
```

57,060 ÷ 509 = 112 ← (Last digit was left unchanged.)

End of Lesson 5

Practical Mathematics
Module 1
Answer Keys

Page 2

1. (a) 12

$10 + 2$	
3×4	
$20 - 8$	
Other answers acceptable	

(b) 9

$3 + 6$	
$63 \div 7$	
$21 - 12$	
Other answers acceptable	

(c) 5×9

45	
$90 \div 2$	
$60 - 15$	
Other answers acceptable	

2. (a) You can make 57 and 75
   (b) You can make 6 three digit numbers, they are 479; 497; 794; 749; 947 and 974.
   (c) 108
   (d) 999
   (e) The counting numbers go on and on without end. For any number that you can name, a greater number can always be obtained by adding 1 to your number. We can never really name the largest number.

Page 6

1.

93,768	773,001
422,930	7,899
209,530	66,002

2. (a) (i) thousands period
   (ii) 6
   (iii) 0 is used as a placeholder

   (b) (i) **comma** between the digits **9** and **0**.
   (ii) 052
   (iii) thousands

Page 7

1. (ii) The digit 6 is in the millions place. It represents the number (**6** × 1 000 000) or **6 000 000**.
   (iii) The digit 8 is in the **ten-thousands** place. It represents the number (**8** × 10 000) or **80 000**.

2. The second number in the chart is **4 600 157**.
   (i) The digit **4** is in the millions place. It represents the number (**4** × 1 000 000) or **4 000 000**.
   (ii) The digits **600** are in the thousands period (Read this one carefully! Think!)
   (iii) The digit **0** is in the thousands place. It represents the number (**0** × 1000) or 0.
   (iv) hundred-thousands. It represents the number (6 × 100 000) or **600 000.**
   (v)

Digit	Value
4	4 000 000
6	600 000
0	0
0	0
1	100
5	50
7	7
	4 600 157

Page 9

1. (i) c (ii) d (iii) c (iv) e (v) b (vi) e
   (vii) b (viii) c (ix) g (x) d

2. (a) thousands or 1000 (c) ones or 1
   (b) tens or 10 (d) hundred-thousands or 100 000

Page 10

3. (a) 10 (b) 100 (c) 10 (d) 1000 (e) 100

2. Second numeral in the chart above is **725 915**
   What digits are in the thousand period? **725**
   In the unit period? **915**

3. Third number in the chart **883 426 912**
   Digit in the billions period none
   In the unit period? **912**
   In the millions period? **883**
   In the thousands period? **426**
   The number is read "883 **million,** 426 **thousand** 912" or "Eight hundred eighty three **million**, four hundred twenty six **thousand**, nine hundred twelve"

4. The fourth numeral in the chart is **584 903 527 661**.
   What digits are in the billions period? **584**
   In the thousands period? **527**
   In the units period? **661**
   In the millions period? **903**
   This number is read: "584 **billion**, 903 **million**, 527 **thousands**, 661.

Page 15

1. (b) two thousand, seventy three
   (c) two hundred ninety-six billion, six hundred seventy-two million, twenty one
   (d) one hundred seventy-seven trillion, two hundred seventy-two billion, six hundred nine million, one hundred seventeen thousand, two hundred
   (e) Seventy six million, nine hundred ninety eight thousand, two hundred three
   (f) Four hundred twenty two billion, nine hundred ninety three million, three thousand, two hundred seventy-seven

Page 16

2. (b) 30 420 200 001
   (c) 78 000 010
   (d) 802 542 000
   (e) 60 412 040
   (f) 200 200 002 020

3. (b) two hundred forty-five million
   (c) seven million
   (d) six hundred four (units)
   (e) eighty eight million
   (f) zero hundreds
   (g) zero ten thousands

4. (a) 3 792 947
   (b) 3 802 847
   (c) 4 492 847
   (d) 792 847
   (e) 1 990
   (f) 1 989

Page 3 **Exercise - Basic Addition Facts**

1. (a) 10 (b) 13 (c) 14 (d) 9
   (e) 16 (f) 14 (g) 17 (h) 12
   (i) 1 (j) 15 (k) 16 (l) 15
   (m) 17 (n) 12 (o) 13 (p) 0
   (q) 7 (r) 9 (s) 11 (t) 12
   (u) 7 (v) 4 (w) 9 (x) 3

2. (a) addend (b) sum (c) sum
   (d) addend (e) addend (f) addend

3. (a) sum
   (b) In the number sentence 3 + 4 = 7, the number **3** and **4** are addends whose **sum** is **7**.
   (c) In the number sentence 7 + 2 = 9, 7 and 2 are **addends** and the number 9 is the **sum**.

   Zero as an Addend
   (a) 3 (b) 9 (c) 7 (d) 6
   (e) 5 + 0 = 5 (f) 0 + 7 = 7 (g) 2 + 0 = 2

   What was the common addend in each exercise? **zero**

   Did you notice any relationship between the remaining addend and the sum?
   Where one addend is zero, the sum is the same as the remaining addend.

Page 5 **Exercise - Commutative Property of Addition**

1. 7
2. 59
3. 923
4. 1768
5. 778
6. 11 789

Page 7

1. (b) $1 + 9 + 8$

(i) $1 + 9 + 8 = (1 + 9) + 8$
$= 10 + 8$
$= 18$

(ii) or $1 + 9 + 8 = 1 + (9 + 8)$
$= 1 + 17$
$= 18$

$\therefore$ (therefore) $(1 + 9) + 8 = 1 + (9 + 8)$

(c) $7 + 8 + 2 = 7 + (8 + 2)$
$= 7 + 10$
$= 17$

or $7 + 8 + 2 = (7 + 8) + 2$
$= 15 + 2$
$= 17$

$\therefore 7 + (8 + 2) = (7 + 8) + 2$

(d) $3 + 5 + 5 = 3 + (5 + 5)$
$= 3 + 10$
$= 13$

or $3 + 5 + 5 = (3 + 5) + 5$
$= 8 + 5$
$= 13$

$\therefore 3 + (5 + 5) = (3 + 5) + 5$

2. (b) $2 + 8 + 3 = 13$

group these first

(c) $5 + 5 + 8 = 18$

group these first

(d) $\left.\begin{matrix}8\\2\end{matrix}\right\}$ group these first
7
17

(e) $\left.\begin{matrix}4\\6\end{matrix}\right\}$ group these first
9
19

(f) $\left.\begin{matrix}5\\5\end{matrix}\right\}$ group these first
1
11

Rearranging Addends

$(3 + 4) + 8 = 7 + 8$ $= 15$	$3 + (4 + 8) = 3 + 12$ $= 15$	$(3 + 8) + 4 = 11 + 4$ $= 15$
$(4 + 3) + 8 = 7 + 8$ $= 15$	$3 + (8 + 4) = 3 + 12$ $= 15$	$(8 + 3) + 4 = 11 + 4$ $= 15$
$8 + (3 + 4) = 8 + 7$ $= 15$	$(4 + 8) + 3 = 12 + 3$ $= 15$	$4 + (3 + 8) = 4 + 11$ $= 15$
$8 + (4 + 3) = 8 + 7$ $= 15$	$(8 + 4) + 3 = 12 + 3$ $= 15$	$4 + (8 + 3) = 4 + 11$ $= 15$

Page 9

1. (a) **order**

$6 + 4 = 4 + 6$
$8 + 6 = 6 + 8$

(b) **grouping**

$(9 + 3) + 7 = 9 + (3 + 7)$
$4 + (1 + 5) = (4 + 1) + 5$

(c) **Commutative and associative**

$6 + (8 + 4) = (6 + 4) + 8$
$(4 + 2) + (6 + 8) = (4 + 6) + (2 + 8)$

2. (a) 14 (b) 18 (c) 8 (d) 18
(e) 22 (f) 18 (g) 27 (h) 28
(i) 29 (j) 16 (k) 20

Page 10

1. 22 2. 21 3. 25 4. 28

Page 11 **Exercises - Addition of Two Digits to One Digit**

1. 42	2. 36	3. 29	4. 36
5. 33	6. 66	7. 55	8. 97
9. 35	10. 23	11. 63	12. 46
13. 30	14. 44	15. 45	16. 44

Page 13 **Exercises - Adding Two-Digit Numbers**

1. 176	2. 106	3. 128	4. 98
5. 157	6. 97	7. 73	8. 132

Page 15

1. 4144	2. 3883	3. 3795	4. 5505
5. 1071	6. 2020	7. 1203	8. 1552
9. 8110	10. 3123	11. 12 613	12. 12 475
13. 36 769	14. 48 634	15. 8926	16. 9270

Page 16

1. **subtrahend; minuend.** The difference is 13.
2. 3
3. –
4. **minuend; two; 14.**

Page 18

1. 7	2. 45	3. 82	4. 31	5. 10
6. 101	7. 112	8. 51	9. 11	10. 26
11. 51	12. 26	13. 22	14. 51	15. 12

Page 20

1. 46	2. 89	3. 74	4. 56	5. 38
6. 26	7. 49	8. 26	9. 13	10. 49
11. 88	12. 88	13. 4	14. 9	15. 12

Page 22

1. 344	2. 135	3. 59	4. 167	5. 89
6. 409	7. 84	8. 80	9. 29	10. 77
11. 159	12. 89	13. 148	14. 641	15. 8472
16. 4885	17. 5034	18. 7476	19. 2777	20. 2621

Page 23 **Exercise - Checking Subtraction**

2. 24

3. 79	4. 1338	5. 820	6. 2577

Page 24

1. 1858	2. 8141		
3. 8378	4. 2719		
5. 6915	6. 15 870		
7. 722	8. 28	9. 2925	10. 2180
11. 6909	12. 1108	13. 3297	14. 3502

15.
$$\begin{array}{r} 5082 \\ -\ 1790 \\ \hline 3292 \end{array}$$

Page 3 **Exercise - Basic Multiplicaton Facts**

1. (a) multiplicand (b) product (c) multiplier
   (d) multiplier (e) product (f) multiplicand

2. (a) factors; product
   (b) **multiplicand** **4** and **6**; and **24** is the product
   (c) factors. **2** and the multiplier is **9**. The number **18** is the product.

3. (a) 0 (b) 20 (c) 8 (d) 144 (e) 110
   (f) 18 (g) 84 (h) 9 (i) 0 (j) 27
   (k) 56 (l) 16 (m) 63 (n) 48 (o) 42
   (p) 120 (q) 72 (r) 49 (s) 27 (t) 28

   Zero as a factor
   (a) 0 (b) 0 (c) 0 (d) 0 (e) 0
   (f) 0 (g) 0 (h) 0

Page 5

What was the common factor in each exercise? **0**
What was the common product in each exercise? **0**
Can you think of a general rule that is shown by the examples on page 4? Whenever zero is a factor, the product is always zero!

One as a factor
(a) 9 (b) 12 (c) 10 (d) 7 (e) 6
(f) 8 (g) 5 (h) 4
What was the common factor in each exercise? **one**

Did you notice any relationship between the remaining factor and the product?
The product was always the same as the remaining factor.

Page 6 **Exercise - Commutative Property of Multiplication**

1. 7 4. 23
2. 10 5. 18
3. 6 6. 1727

Page 7

1. (b) $8 \times 1 \times 9 = (8 \times 1) \times 9$ or $8 \times 1 \times 9 = 8 \times (1 \times 9)$
   $= 8 \times 9$ $= 8 \times 9$
   $= 72$ $= 72$

   therefore $(8 \times 1) \times 9 = 8 \times (1 \times 9)$

(c) $10 \times 0 \times 5 = (10 \times 0) \times 5$ or $10 \times 0 \times 5 = 10 \times (0 \times 5)$
$= 0 \times 5$ $= 10 \times 0$
$= 0$ $= 0$

therefore $(10 \times 0) \times 5 = 10 \times (0 \times 5)$

(d) $5 \times 2 \times 11 = (5 \times 2) \times 11$ or $5 \times 2 \times 11 = 5 \times (2 \times 11)$
$= 10 \times 11$ $= 5 \times 22$
$= 110$ $= 110$

therefore $(5 \times 2) \times 11 = 5 \times (2 \times 11)$

(e) $12 \times 3 \times 4 = (12 \times 3) \times 4$ or $12 \times 3 \times 4 = 12 \times (3 \times 4)$
$= 36 \times 4$ $= 12 \times 12$
$= 144$ $= 144$

therefore $(12 \times 3) \times 4 = 12 \times (3 \times 4)$

(f) $9 \times 8 \times 7 = (9 \times 8) \times 7$ or $9 \times 8 \times 7 = 9 \times (8 \times 7)$
$= 72 \times 7$ $= 9 \times 56$
$= 504$ $= 504$

therefore $(9 \times 8) \times 7 = 9 \times (8 \times 7)$

Page 8

$(3 \times 5) \times 2 = 15 \times 2$ $= 30$	$3 \times (5 \times 2) = 3 \times 10$ $= 30$	$(3 \times 2) \times 5 = 6 \times 5$ $= 30$
$(5 \times 3) \times 2 = 15 \times 2$ $= 30$	$3 \times (2 \times 5) = 3 \times 10$ $= 30$	$(2 \times 3) \times 5 = 6 \times 5$ $= 30$
$2 \times (3 \times 5) = 2 \times 15$ $= 30$	$(5 \times 2) \times 3 = 10 \times 3$ $= 30$	$5 \times (3 \times 2) = 5 \times 6$ $= 30$
$2 \times (5 \times 3) = 2 \times 15$ $= 30$	$(2 \times 5) \times 3 = 10 \times 3$ $= 30$	$5 \times (2 \times 3) = 5 \times 6$ $= 30$

Page 9

1. (a) order; product; $3 \times 9 = 9 \times 3$ $8 \times 4 = 4 \times 8$
   (b) grouping; product; $(1 \times 6) \times 9 = 1 \times (6 \times 9)$ $4 \times (3 \times 7) = (4 \times 3) \times 7$
   (c) commutative; associative; $6 \times (5 \times 4) = (5 \times 6) \times 4$
   (d) zero (or 0) $(3 \times 9) \times (4 \times 6) = (3 \times 4 \times (9 \times 6)$

2. (a) 72 (b) 56 (c) 0 (d) 24 (e) 240 (f) 0

Page 11

Break 215 into units, tens, hundreds and then apply the distributive property.

$$\begin{aligned} 6 \times 215 &= 6(200 + 10 + 5) \\ &= (6 \times 200) + (6 \times 10) + (6 \times 5) \\ &= 1200 + 60 + 30 \\ &= 1290 \end{aligned}$$

Exercise - Using the Distributive Property

1. (a) $7 \times 14 = 7 \times (10 + 4) = (7 \times 10) + (7 \times 4) = 70 + 28 = \mathbf{98}$

(b) $\begin{aligned} 9 \times 281 &= 9 \times (200 + 80 + 1) = (9 \times 200) + (9 \times 80) + (9 \times 1) \\ &= 1800 + 720 + 9 \\ &= \mathbf{2529} \end{aligned}$

(c) $\begin{aligned} 2 \times 193 &= 2 \times (100 + 90 + 3) \\ &= (2 \times 100) + (2 \times 90) + (2 \times 3) \\ &= 200 + 180 + 6 \\ &= \mathbf{386} \end{aligned}$

(d) $\begin{aligned} 5 \times 513 &= 5 \times (500 + 10 + 3) \\ &= (5 \times 500) + (5 \times 10) + (5 \times 3) \\ &= 2500 + 50 + 15 \\ &= \mathbf{2565} \end{aligned}$

(e) $\begin{aligned} 6 \times 918 &= 6 \times (900 + 10 + 8) \\ &= (6 \times 900) + (6 \times 10) + (6 \times 8) \\ &= 5400 + 60 + 48 \\ &= \mathbf{5508} \end{aligned}$

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Exercise - Multiplying Several Digits by One Digit

1. 850	2. 1608	3. 2748	4. 59 801
5. 24 102	6. 22 910	7. 957	8. 2688
9. 6728	10. 29 248	11. 19 434	12. 47 097
13. 1395	14. 4130	15. 54 810	16. 174 717

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2.
$$\begin{array}{r} 86 \\ \times\ 37 \\ \hline 602 \\ 258 \\ \hline 3182 \end{array}$$

3.
$$\begin{array}{r} 53 \\ \times\ 11 \\ \hline 53 \\ 53 \\ \hline 583 \end{array}$$

4.
$$\begin{array}{r} 38 \\ \times\ 21 \\ \hline 38 \\ 76 \\ \hline 798 \end{array}$$

5.
$$\begin{array}{r} 76 \\ \times\ 52 \\ \hline 152 \\ 380 \\ \hline 3952 \end{array}$$

6.
$$\begin{array}{r} 61 \\ \times\ 94 \\ \hline 244 \\ 549 \\ \hline 5734 \end{array}$$

7.
$$\begin{array}{r} 42 \\ \times\ 83 \\ \hline 126 \\ 336 \\ \hline 3486 \end{array}$$

8.
$$\begin{array}{r} 25 \\ \times\ 59 \\ \hline 225 \\ 125 \\ \hline 1475 \end{array}$$

9.
$$\begin{array}{r} 356 \\ \times\ 24 \\ \hline 1424 \\ 712 \\ \hline 8544 \end{array}$$

10.
$$\begin{array}{r} 3175 \\ \times\ 84 \\ \hline 12700 \\ 25400 \\ \hline 266700 \end{array}$$

11.
$$\begin{array}{r} 5169 \\ \times\ 45 \\ \hline 25845 \\ 20676 \\ \hline 232605 \end{array}$$

12.
$$\begin{array}{r} 7032 \\ \times\ 93 \\ \hline 21096 \\ 63288 \\ \hline 653976 \end{array}$$

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2.
$$\begin{array}{r} 442 \\ \times\ 623 \\ \hline 1326 \\ 884 \\ 2652 \\ \hline 275366 \end{array}$$

3.
$$\begin{array}{r} 753 \\ \times\ 175 \\ \hline 2259 \\ 5271 \\ 753 \\ \hline 130269 \end{array}$$

4.
$$\begin{array}{r} 902 \\ \times\ 466 \\ \hline 5412 \\ 5412 \\ 3608 \\ \hline 420332 \end{array}$$

5.
$$\begin{array}{r} 5643 \\ \times\ 152 \\ \hline 11286 \\ 28215 \\ 5643 \\ \hline 857736 \end{array}$$

6.
$$\begin{array}{r} 4071 \\ \times\ 389 \\ \hline 36639 \\ 32568 \\ 12213 \\ \hline 1583619 \end{array}$$

7.
$$\begin{array}{r} 928 \\ \times\ 576 \\ \hline 5568 \\ 6496 \\ 4640 \\ \hline 534528 \end{array}$$

8.
$$\begin{array}{r} 5833 \\ \times\ 979 \\ \hline 52497 \\ 40831 \\ 52497 \\ \hline 5710507 \end{array}$$

9.
$$\begin{array}{r} 7652 \\ \times\ 219 \\ \hline 68868 \\ 7652 \\ 15304 \\ \hline 1675788 \end{array}$$

10.
$$\begin{array}{r} 7238 \\ \times\ 526 \\ \hline 43428 \\ 14476 \\ 36190 \\ \hline 3807188 \end{array}$$

11.
$$\begin{array}{r} 2192 \\ \times\ 317 \\ \hline 15344 \\ 2192 \\ 6576 \\ \hline 694864 \end{array}$$

12.
$$\begin{array}{r} 5301 \\ \times\ 272 \\ \hline 10602 \\ 37107 \\ 10602 \\ \hline 1441872 \end{array}$$

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1. (a) one zero
   (b) two zeros
   (c) three zeros
   (d) 8; two zeros

2. (b)
$$\begin{array}{r} 1500 \\ \times\ 60 \\ \hline 90000 \end{array}$$
6×15 attach 3 zeros

(c)
$$\begin{array}{r} 1130 \\ \times\ 20 \\ \hline 22600 \end{array}$$
2×113 attach 2 zeros

(d)
$$\begin{array}{r} 1000 \\ \times\ 100 \\ \hline 100000 \end{array}$$
attach 5 zeros

(e)
$$\begin{array}{r} 17 \\ \times\ 200 \\ \hline 3400 \end{array}$$
2×17 attach 2 zeros

(f)
$$\begin{array}{r} 80 \\ \times\ 100 \\ \hline 8000 \end{array}$$

(g)
$$\begin{array}{r} 680 \\ \times\ 100 \\ \hline 68000 \end{array}$$

(h)
$$\begin{array}{r} 521 \\ \times\ 10000 \\ \hline 5210000 \end{array}$$
attach 4 zeros

(i)
$$\begin{array}{r} 3004 \\ \times\ 51 \\ \hline 3004 \\ 15020 \\ \hline 153204 \end{array}$$

(j)
$$\begin{array}{r} 702 \\ \times\ 301 \\ \hline 702 \\ 21060 \\ \hline 211302 \end{array}$$

(k)
$$\begin{array}{r} 1903 \\ \times\ 516 \\ \hline 11418 \\ 1903 \\ 9515 \\ \hline 981948 \end{array}$$

(l)
$$\begin{array}{r} 1802 \\ \times\ 7001 \\ \hline 1802 \\ 12614 \\ \hline 12615802 \end{array}$$

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1. (a)
$$\begin{array}{r} 38 \\ \times\ 86 \\ \hline 228 \\ 304 \\ \hline 3268 \end{array}$$

(b)
$$\begin{array}{r} 67 \\ \times\ 35 \\ \hline 335 \\ 201 \\ \hline 2345 \end{array}$$

(c)
$$\begin{array}{r} 268 \\ \times\ 36 \\ \hline 1608 \\ 804 \\ \hline 9648 \end{array}$$

(d)
$$\begin{array}{r} 468 \\ \times\ 502 \\ \hline 936 \\ 2340 \\ \hline 234936 \end{array}$$

2. (a)
$$\begin{array}{r} 7032 \\ \times\ 93 \\ \hline 21096 \\ 63288 \\ \hline 653976 \end{array}$$

(b)
$$\begin{array}{r} 5169 \\ \times\ 45 \\ \hline 25845 \\ 20676 \\ \hline 232605 \end{array}$$

(c)
$$\begin{array}{r} 9274 \\ \times\ 26 \\ \hline 55644 \\ 18548 \\ \hline 241124 \end{array}$$

(d)
$$\begin{array}{r} 5480 \\ \times\ 71 \\ \hline 5480 \\ 38360 \\ \hline 389080 \end{array}$$

(e)
$$\begin{array}{r} 356 \\ \times\ 24 \\ \hline 1424 \\ 712 \\ \hline 8544 \end{array}$$

(f)
$$\begin{array}{r} 2090 \\ \times\ 56 \\ \hline 12540 \\ 10450 \\ \hline 117040 \end{array}$$

(g)
$$\begin{array}{r} 4286 \\ \times\ 93 \\ \hline 12858 \\ 38574 \\ \hline 398598 \end{array}$$

(h)
$$\begin{array}{r} 430 \\ \times\ 27 \\ \hline 3010 \\ 860 \\ \hline 11610 \end{array}$$

(i)
$$\begin{array}{r} 9134 \\ \times\ 56 \\ \hline 54804 \\ 45670 \\ \hline 511504 \end{array}$$

(j)
$$\begin{array}{r} 493 \\ \times\ 29 \\ \hline 4437 \\ 986 \\ \hline 14297 \end{array}$$

(k)
$$\begin{array}{r} 5678 \\ \times\ 87 \\ \hline 39746 \\ 45424 \\ \hline 493986 \end{array}$$

(l)
$$\begin{array}{r} 9876 \\ \times\ 91 \\ \hline 9876 \\ 8884 \\ \hline 898716 \end{array}$$

(m)
$$\begin{array}{r} 516 \\ \times\ 54 \\ \hline 2064 \\ 2580 \\ \hline 27864 \end{array}$$

(n)
$$\begin{array}{r} 18702 \\ \times\ 108 \\ \hline 149616 \\ 18702 \\ \hline 2019816 \end{array}$$

(o)
$$\begin{array}{r} 29080 \\ \times\ 100 \\ \hline 2908000 \end{array}$$

1. (a) **48** is the dividend; **8** is the divisor; and 6 is the quotient.
   (b) the number 8 is the **quotient**; 9 is the **divisor** and the number 72 is the **dividend.**
   (c) $36 \div 12 = 3$
   Is there a remainder? **No**

2. (b) $19 \div 6 = 3$ r 1
   ↑
   divisor

   (c) $8\overline{)32}$ $\quad 4$
   $\underline{32}$
   0
   ↑
   remainder

   (d) $49 \div 7 = 7$
   ↑
   dividend

   (e) $81 \div 9 = 9$ ← **divisor**

   (f) $56 \div 7 = 8$ ← **quotient**

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2. $4\overline{)724}$ $\quad 181$
   $\underline{4}$
   32
   $\underline{32}$
   04
   $\underline{4}$
   0

3. $3\overline{)921}$ $\quad 307$
   $\underline{9}$
   021
   $\underline{21}$
   0

4. $2\overline{)829}$ $\quad 414$
   $\underline{8}$
   02
   $\underline{2}$
   09
   $\underline{8}$
   1

5. $7\overline{)287}$ $\quad 41$
   $\underline{28}$
   07
   $\underline{7}$
   0

6. $9\overline{)819}$ $\quad 91$
   $\underline{81}$
   09
   $\underline{9}$
   0

7. $8\overline{)328}$ $\quad 41$
   $\underline{32}$
   08
   $\underline{8}$
   0

8. $3\overline{)936}$ $\quad 312$
   $\underline{9}$
   03
   $\underline{3}$
   06
   $\underline{6}$
   0

9. $5\overline{)955}$ $\quad 191$
   $\underline{5}$
   45
   $\underline{45}$
   05
   $\underline{5}$
   0

10. $7\overline{)608}$ = 86, 56, 48, 42, 6

11. $6\overline{)186}$ = 31, 18, 006, 6, 0

12. $7\overline{)4977}$ = 711, 49, 007, 7, 07, 7, 0

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2. $24\overline{)1776}$ = 74, 168, 96, 96, 0

3. $55\overline{)3276}$ = 59, 275, 526, 495, 31

4. $41\overline{)2634}$ = 74, 246, 174, 164, 10

5. $37\overline{)2963}$ = 80, 296, 03, 0, 3

6. $98\overline{)8541}$ = 87, 784, 701, 686, 15

7. $18\overline{)4507}$ = 250, 36, 90, 90, 07, 0, 7

8. $19\overline{)1672}$ = 88, 152, 152, 152, 0

9. $94\overline{)9212}$ = 98, 846, 752, 752, 0

10. $91\overline{)873621}$ = 9600, 819, 546, 546, 021

11. $341\overline{)8476321}$ = 24857, 682, 1656, 1364, 2923, 2728, 1952, 1705, 2471, 2387, 84

12. $17\overline{)38692130}$ = 2276007, 34, 46, 34, 129, 119, 102, 102, 0130, 119, 11

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1. $450 \div 9 = 50$
2. $640 \div 8 = 80$
3. $560 \div 8 = 70$
4. $300 \div 5 = 60$
5. $25\,000 \div 500 = 50$
6. $360 \div 90 = 4$
7. $3600 \div 60 = 60$
8. $18\,000 \div 30 = 600$

Page 13 **Exercises - Finding the Missing Factor**

1. 3 2. 11 3. 7 4. 21 5. 47

Page 14

1. (b)
$$\begin{array}{r} 402 \\ 24\overline{)9648} \\ \underline{96} \\ 048 \\ \underline{48} \\ 0 \end{array}$$

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1. (c)
$$\begin{array}{r} 210 \\ 96\overline{)20160} \\ \underline{192} \\ 96 \\ \underline{96} \\ 00 \\ \underline{0} \\ 0 \end{array}$$

(d)
$$\begin{array}{r} 203 \\ 20\overline{)4060} \\ \underline{40} \\ 060 \\ \underline{60} \\ 0 \end{array}$$

2. (b)
$$\begin{array}{r} 40 \\ 78\overline{)3176} \\ \underline{312} \\ 56 \\ \underline{0} \\ 56 \end{array}$$

(c)
$$\begin{array}{r} 48 \\ 34\overline{)1635} \\ \underline{136} \\ 275 \\ \underline{272} \\ 3 \end{array}$$

(d)
$$\begin{array}{r} 570 \\ 16\overline{)9123} \\ \underline{80} \\ 112 \\ \underline{112} \\ 03 \\ \underline{0} \\ 3 \end{array}$$

3. (a)
$$\begin{array}{r} 249 \\ 319\overline{)79635} \\ 638 \\ \hline 1583 \\ 1276 \\ \hline 3075 \\ 2871 \\ \hline 204 \end{array}$$

(b)
$$\begin{array}{r} 319 \\ \times\ 249 \\ \hline 2871 \\ 1276 \\ 638 \\ \hline 79431 \\ +\ 204 \\ \hline 79835 \end{array}$$

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1. (a) zero (b) impossible (c) dividend (d) divisor (e) zero

2. (a) 0 (b) 0
(c) impossible (d) impossible
(e) impossible (f) 0
(g) 0 (h) 0
(i) impossible (j) impossible

Note: You may use the symbol ϕ to indicate impossible. ϕ - read as "phi"

3. (a) 0 (b) 0
(c) any number (d) any non-zero number
(e) 0 (f) 0

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1.
$$\begin{array}{r} 190 \\ 7 \\ 12 \\ 26 \\ 2 \\ +100 \\ \hline 337 \end{array}$$

The total weight is 337 kg.

3. $36 \times 12 =$
$36 \times 12 = 432$ minutes

He will take 432 minutes
(or 7 hours 12 minutes)

4. $3945 \div 5 =$

$$\begin{array}{r} 789 \\ 5\overline{)3945} \\ \underline{35} \\ 44 \\ \underline{40} \\ 45 \\ \underline{45} \\ 0 \end{array}$$

He must average 789 km per day.

5. (a) $128 + 46 + 51 + 36 + 40 + 13 =$

$$\begin{array}{r} 128 \\ 46 \\ 51 \\ 36 \\ 40 \\ +\ 13 \\ \hline 314 \end{array}$$

He used 314 feet of wire.

(b) $600 - 314 =$

$$\begin{array}{r} 600 \\ -314 \\ \hline 286 \end{array}$$

Amount left is 286 feet.

6. Time required for each door stopper would be (20 + 1) minutes or 21 minutes.

Therefore the time taken to make 120 door stoppers would be: (120 × 21) minutes.

$$\begin{array}{r} 120 \\ \times\ 21 \\ \hline 120 \\ 240 \\ \hline 2520 \end{array}$$

So the time needed to make these stoppers is 2520 minutes. We must now change this 2520 minutes into hours.

$2520 \div 60 =$

$$\begin{array}{r} 42 \\ 60\overline{)2520} \\ \underline{240} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

The 120 door stoppers would take 42 hours to make.

At $3.00 per hour the total cost would be:

$42 \times \$3$ or

$$\begin{array}{r} 42 \\ \times\ 3 \\ \hline \$\ 126 \end{array}$$

Conclusion: The total labour cost for the door stoppers would be $126.

Page 3

Rounding to the nearer thousand

If you round a number to the nearer thousand, the digit to look at is in the **hundreds** place in the numberal. If the value of the digit is five or **greater** then replace the **thousands** digit with the next greater digit, placing zeros in the **hundreds, tens** and **ones** places. If the value of the hundreds digit is less than **five** leave the **thousands** digit unchanged and place the **zeros** in the hundreds, tens and ones places.

Page 6

1.

		Rounded to Nearer ten	Rounded to Nearer hundred	Rounded to Nearer thousand
1.	1739	1 740	1 700	2 000
2.	20 735	20 740	20 700	21 000
3.	91 244	91 240	91 200	91 000
4.	5 760	5 760	5 800	6 000
5.	14 526	14 530	14 500	15 000
6.	9 991	9 990	10 000	10 000
7.	8 723	8 720	8 700	9 000
8.	45 628	45 630	45 600	46 000
9.	723 401	723 400	723 400	723 000
10.	197 259	197 260	197 300	197 000
11.	1 804 523	1 804 520	1 804 500	1 805 000
12.	9 478 235	9 478 240	9 478 200	9 478 000

2. To the nearer hundred

3. To the nearer hundred.

4. (a)
$$\begin{array}{r} 5307 \\ 18\overline{)95535} \\ \underline{90} \\ 55 \\ \underline{54} \\ 135 \\ \underline{126} \\ 9 \end{array}$$

Answer $5307\frac{9}{18}$ – This $\frac{9}{18}$ is exactly half of the divisor, as a convention we round up to the next whole number. So rounded number is 5308.

(b)
$$\begin{array}{r} 301 \\ 207\overline{)62409} \\ \underline{621} \\ 309 \\ \underline{207} \\ 102 \end{array}$$

Answer $301\frac{102}{207}$ – as fraction is less than half. We round down to the nearest whole number. So rounded number is 301.

5. Time taken is $2045 \div 90$

$$\begin{array}{r} 22 \\ 90\overline{)2045} \\ \underline{180} \\ 245 \\ \underline{180} \\ 65 \end{array}$$

Answer is $22\frac{65}{90}$ – consequently we round up (as $\frac{65}{90} > \frac{1}{2}$)

So time needed for the journey would be **23 hours.**

6. Equation would be $825\,432 \div 5$

$$\begin{array}{r} 165086 \text{ r}2 \\ 5\overline{)825432} \\ \underline{5} \\ 32 \\ \underline{30} \\ 25 \\ \underline{25} \\ 043 \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

Each child should receive $165 086.

9

1. Total number would be $10 + 38 + 22 + 46 + 24 + 81 + 35 = 256$

There are 11 students, so average is $\frac{\text{Total age in years}}{\text{Number of students}} = \frac{256}{11} = 23$ (to the nearer whole number)

Concluding statement – The average of the students was 23 years.

2. Total of Steve's grades $= 83 + 76 + 82 + 71 + 89 + 77 + 82$
$= 560$

Steve's average $= \frac{560}{7} = 80\%$

So Steve will not have to write the examination.

3.

Province	Average Acres per Farm
Newfoundland	21
Prince Edward Island	113
Nova Scotia	132
New Brunswick	135
Quebec	130
Ontario	141
Manitoba	364
Saskatchewan	607
Alberta	579
British Columbia	183
Territories	204

The answer to these questions have been rounded to the nearest whole number (unit).

4. Total number of men is 117, total units is 432.

$$\text{Average} = \frac{\text{Total units}}{\text{Total men}} = \frac{432}{117} = 3 \text{ remainder } 81$$

or $3\frac{81}{117}$ which would round up to 4 units per man.

Concluding statement: The average number of units is 4 units per man per week.

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1. (b) 19 $1 \times 19 = 19$
Factors are 1 and 19.

(c) 38
$1 \times 38 = 38$
$2 \times 19 = 38$
Factors are 1, 2, 19 and 38.

(d) 49
$1 \times 49 = 49$
$7 \times 7 = 49$
Factors are 1, 7 and 49.

(e) 75
$1 \times 75 = 75$
$3 \times 25 = 75$
$5 \times 15 = 75$
Factors are 1, 3, 5, 15, 25 and 75.

(f) 36
$1 \times 36 = 36$
$2 \times 18 = 36$
$3 \times 12 / 36$
$4 \times 9 = 36$
$6 \times 6 = 36$
Factors are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

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1. (a) 35 composite $35 = 5 \times 7$
   (b) 23 prime
   (c) 26 composite $2 \times 13 = 26$
   (d) 57 composite $3 \times 19 = 57$
   (e) 58 composite $2 \times 29 = 58$
   (f) 59 prime
   (g) 71 prime
   (h) 33 composite $3 \times 11 = 33$
   (i) 73 prime
   (j) 101 prime

2. (a) 210
   $= 2 \times 105$
   $= 2 \times (3 \times 35)$
   $= 2 \times 3 \times 5 \times 7$

   (b) 22
   $= 2 \times 11$

   (c) 60
   $= 2 \times 30$
   $= 2 \times (2 \times 15)$
   $= 2 \times 2 \times 3 \times 5$

   (d) 36
   $= 2 \times 18$
   $= 2 \times (2 \times 9)$
   $= 2 \times 2 \times 3 \times 3$

   (e) 84
   $= 2 \times 42$
   $= 2 \times 2 \times 21$
   $= 2 \times 2 \times 3 \times 7$

   (f) 92
   $= 2 \times 46$
   $= 2 \times 2 \times 23$

   (g) 380
   $= 2 \times 190$
   $= 2 \times 2 \times 95$
   $= 2 \times 2 \times 5 \times 19$

   (h) 712
   $= 2 + 356$
   $= 2 \times 2 \times 178$
   $= 2 \times 2 \times 2 \times 89$

   (i) 195
   $= 3 \times 65$
   $= 3 \times 5 \times 13$

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1. 18, 24
   $18 = \mathbf{2} \times 3 \times \mathbf{3}$
   $24 = \mathbf{2} \times 2 \times 2 \times \mathbf{3}$

   H.C.F. $= 2 \times 3 = 6$

2. 60, 96
   $60 = \mathbf{2} \times \mathbf{2} \times 3 \times 5$
   $96 = \mathbf{2} \times \mathbf{2} \times 2 \times 2 \times 2 \times \mathbf{3}$

   H.C.F. $= 2 \times 2 \times 3 = 12$

3. 72, 63
   $72 = 2 \times 2 \times 2 \times \mathbf{3} \times \mathbf{3}$
   $63 = \mathbf{3} \times \mathbf{3} \times 7$

   H.C.F. $= 3 \times 3 = 9$

4. 36, 54
   $36 = \mathbf{2} \times 2 \times \mathbf{3} \times \mathbf{3}$
   $54 = \mathbf{2} \times 3 \times \mathbf{3} \times \mathbf{3}$

   H.C.F. $= 2 \times 3 \times 3 = 18$

5. 28, 70
   $28 = 2 \times 2 \times 7$
   $70 = 2 \times 5 \times 7$

   H.C.F. $= 2 \times 7 = 14$

6. 18, 76, 95
   $18 = 2 \times 3 \times 3$
   $76 = 2 \times 2 \times 19$
   $95 = 5 \times 19$
   This is a 'tricky' question. There is no number that is a factor of all three given numbers. In this case the H.C.F. will be 1 (one).

7. 24, 36, 56
   $24 = 2 \times 2 \times 2 \times 3$
   $36 = 2 \times 2 \times 3 \times 3$
   $56 = 2 \times 2 \times 2 \times 7$

   H.C.F. $= 2 \times 2 = 4$

8. 64, 92, 128, 184
   $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
   $92 = 2 \times 2 \times 23$
   $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
   $184 = 2 \times 2 \times 2 \times 23$

   H.C.F. $= 2 \times 2 = 4$

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Is 12 a multiple of 4? **Yes** Is 18 a multiple of 4? **No**
Is 5 a multiple of 10? **No** Is 96 a multiple of 8? **Yes**

Find the first five multiples of 7 **7 14 21 28 35**
Find the first four multiples of 11 **11 22 33 44**

Find a number that is a multiple of both 2 and 3 **6**

The number 30 is a multiple of what numbers. **1, 2, 3, 5, 6, 10, 15, 30**

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1. Multiples of 9 are 9, 18, 27, **36, 45,** 54, ...
   Multiples of 15 are 15, 30, **45, 60,** 75, ...
   The L.C.M. is **45.**

2. 3, 6 and 8
   Multiples of 3 are **3, 6,** 9, **12, 15,** 18, **21, 24,** 27, ...
   Multiples of 6 are 6, **12,** 18, **24,** 30, **36,** ...
   Multiples of 8 are **8,** 16, **24, 32, 40, ...**
   The L.C.M. is **24.**

3. 10 and 12
   Multiples of 10 are **10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, ...**
   Multiples of 12 are **12, 24, 36, 48, 60, 72, 84, ...**
   The L.C.M. is **60.**

Note: The L.C.M. of two numbers is the smallest number that those two numbers will divide into evenly.
Example: The L.C.M. of 9 and 15 is 45 because 45 is the smallest number that 9 and 15 will divide into evenly.

Page 20 **Exercise - Lowest Common Multiple**

2. 4, 6, 10
   $4 = 2 \times 2$
   $6 = 3 \times 2$
   $10 = 5 \times 2$
   Lowest common multiple $= \mathbf{2} \times \mathbf{2} \times 3 \times 5$
   $= 60$

   Since 2 appears twice in the factorization of 4, it must appear twice in the L.C.M.

3. 5, 6, 9
   $5 = 1 \times 5$ (prime number)
   $6 = 3 \times 2$
   $9 = 3 \times 3$

   L.C.M. $= 5 \times 3 \times 3 \times 2$
   $= 90$

4. 4, 8, 16
   $4 = 2 \times 2$
   $8 = 2 \times 2 \times 2$
   $16 = 2 \times 2 \times 2 \times 2$

   L.C.M. $= 2 \times 2 \times 2 \times 2$
   $= 16$

5. 3, 15, 18
   $3 = 1 \times 3$ (prime number)
   $15 = 3 \times 5$
   $18 = 3 \times 3 \times 2$

   L.C.M. $= 3 \times 3 \times 5 \times 2$
   $= 90$

6. 8, 10, 14
   $8 = 2 \times 2 \times 2$
   $10 = 5 \times 2$
   $14 = 7 \times 2$

   L.C.M. $= 2 \times 2 \times 2 \times 5 \times 7$
   $= 280$

7. 3, 7, 21
   $3 = 3 \times 1$ (prime)
   $7 = 7 \times 1$ (prime)
   $21 = 3 \times 7$

   L.C.M. $= 3 \times 7 = 21$

8. 60, 12, 3
   $60 = 2 \times 2 \times 3 \times 5$
   $12 = 2 \times 2 \times 3$
   $3 = 1 \times 3$ (prime)

   L.C.M. $= 2 \times 2 \times 3 \times 5 = 60$

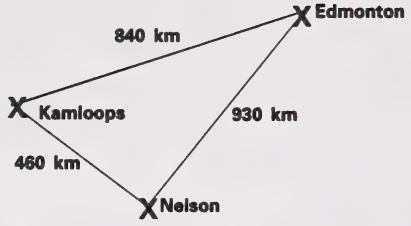
9. 9, 5
   $9 = 3 \times 3$
   $5 = 5 \times 1$ (prime)
   L.C.M. $= 3 \times 3 \times 5$

10. 7, 49, 149
    $7 = 7 \times 1$ (prime)
    $49 = 7 \times 7$
    $149 = 1 \times 149$ (prime)

    L.C.M. $= 7 \times 7 \times 149$
    $= 7301$

2. In this kind of question, it is sometime beneficial to draw a diagram or sketch of the problem, it will help the student visualize what is happening, and therefore help you solve the problem.

Sketch



(a) He travelled from Edmonton to Nelson and then on to Kamloops.

For an estimated answer we should round to the nearest 100 km.

	Rounded
Edmonton to Nelson 930 km	900 km
Nelson to Kamloops 460 km	500
Total distance would be approximately	1400 km

(b) If he had driven directly the distance would have been 800 km (to the nearest hundred)

So he drove	1400 (route via Nelson)
	− 800 (direct route)
He drove an extra 600 km.	600 km

3. Round \$118.50 to \$120.00.

The approximate cost of each tire would be \$120 ÷ 4 = \$30.

4. (a)

\$162 rounds to	160
8 rounds to	10
46 rounds to	50
12 rounds to	10
108 rounds to	110
17 rounds to	20
36 rounds to	40
24 rounds to	20
	\$420

The total expenses per month are about \$420.

(b) Round her total salary from \$728 to \$720. So she can save approximately

720
−420
\$300

Conclusion: She can save about \$300 per month.

5. Number of bottles = $12 \times 12 = 144$ Rounds to 140
   Number of people = 23 Rounds to 20

   So approximate number of bottles per person would be $140 \div 20 = 7$

   So each person could have approximately 7 bottles each.
   (Note – many other estimates would also be acceptable.)

N.L.C. - B.N.C.
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